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Status of the Superworld
From Theory to Experiment

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ABSTRACT

Among the most outstanding conceptual developments in particle physics we have: the unification of all particle interactions at very-high energies (Grand Unification), the fermion-boson symmetry (Supersymmetry), the non-point-like structure of elementary particles (String theory), and the understanding that all dynamical quantities (gauge couplings, masses, Yukawa couplings) run with energy (Renormalization Group Equations). The goal is to make use of these great developments to construct a theory which embraces all fundamental forces of Nature, including gravity. In this review we address this problem and its possible implications for physics in the energy range where our experimental facilities operate. We show that what is required are not qualitative arguments but a set of detailed calculations with definite predictions. This is why we have chosen two specific Supergravity models: $SU(5)$ and $SU(5)\times U(1)$. One as representative of Field Theory, the other of String Theory. This kind of model-building at the Planck scale seems to tell us that new physics beyond the Standard Model could be near the Fermi scale. Therefore detailed calculations for existing facilities (Tevatron, LEP I-II, HERA, Gran Sasso, Super Kamiokande, Dumand, Amanda, etc.) are a definite way to put string theory – *i.e.*, the existence of the Superworld – under experimental test, now.

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1 Introduction

To unify all fundamental forces of Nature is the dream of most physicists. Important developments in this direction have taken place during the last two decades, namely grand unification, supersymmetry, and string theory. The first predicts that all particle interactions become of equal strength above a very high energy scale, the second puts bosons and fermions on equal footing, and the third abandons the point-like structure of elementary particles in favour of one-dimensional "strings". A nice feature of string theory is that it needs to be supersymmetric, thus suggesting that the Superworld may really exist and be discovered soon, if supersymmetry is effectively broken at low energies.

The advent of LEP has allowed to measure the three gauge couplings of the Standard Model, $SU(3)_C$, $SU(2)_L$, $U(1)_Y$ (at M_Z) with unprecedented accuracy (see *e.g.*, Ref. [1]):

$$\begin{aligned}\alpha_1^{-1} &= 58.83 \pm 0.11,\\ \alpha_2^{-1} &= 29.85 \pm 0.11,\\ \alpha_3^{-1} &= 8.47 \pm 0.57.\end{aligned}$$

Furthermore, the number of families has been determined to be [1]

$$N_f = 2.980 \pm 0.027.$$

From these experimental data the Renormalization Group Equations (RGEs) allow to span as many orders of magnitude as wanted, provided we know the virtual phenomena to be accounted for in these large ranges of energy, starting at M_Z.

The key features in this field of physics are therefore the quantities above and the RGEs. From these ingredients we would like to know if it is possible to predict the threshold for the lightest detectable supersymmetric particle. It is a field which we have studied since the late 70's. We realized the importance of the new degree of freedom (the threshold for supersymmetric particle production) introduced in the running of the gauge couplings $(\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1})$ for their convergence towards a unique value. However, we also noticed the many problems to be overcome in the understanding of where the energy level for the superparticles could be. This review paper is organized as follows. In Section 2 the problem of the convergence of the gauge couplings $\alpha_1, \alpha_2, \alpha_3$ is discussed. In Section 3 the constraints from the unification conditions are presented in order to allow the reader an understanding of the physical consequences hidden in the mathematical formalism. Section 4 discusses the popular quantity M_{SUSY} and the reasons why its use should be discontinued. Section 5 presents the need to promote global supersymmetry to be a local one thus giving rise to the Supergravity description of low-energy phenomena. Sections 6 and 7 present the two Supergravity Models $SU(5)$ and $SU(5) \times U(1)$ and in Sections 8, 9, 10, 11 detailed calculations for the Tevatron, LEP I-II, HERA, and the Underground Laboratories plus Underwater Facilities are reported. Section 12 is devoted to detailed calculations for indirect experimental detection. In Section 13 the origin of mass

problem is addressed together with a working ground to predict m_t. Our conclusions are summarized in Section 14.

2 High precision LEP data and convergence of couplings: physics is not Euclidean geometry

Experimentally we know that (see *e.g.*, [2]),

$$\alpha_e^{-1}(M_Z) = 127.9 \pm 0.2, \quad (1)$$
$$\alpha_3(M_Z) = 0.118 \pm 0.008, \quad (2)$$
$$\sin^2\theta_W(M_Z) = 0.2334 \pm 0.0008. \quad (3)$$

The $U(1)_Y$ and $SU(2)_L$ gauge couplings are related to these by $\alpha_1 = \frac{5}{3}(\alpha_e/\cos^2\theta_W)$ and $\alpha_2 = (\alpha_e/\sin^2\theta_W)$. As mentioned above, the values of the three gauge couplings of the Standard Model derived from Eqs. (1),(2), and (3), are:

$$\text{at} \quad Q = M_Z \qquad \begin{cases} \alpha_1^{-1} = 58.83 \pm 0.11 \\ \alpha_2^{-1} = 29.85 \pm 0.11 \\ \alpha_3^{-1} = 8.47 \pm 0.57 \end{cases} \quad (4)$$

These three gauge couplings evolve with increasing values of the scale Q in a logarithmic fashion, and may become equal at some higher scale, signaling the possible presence of a larger gauge group. However, this need not be the case: the three gauge couplings may meet and then depart again. Conceptually, the presence of a unified group is essential in the discussion of unification of couplings. In this case, the newly excited degrees of freedom will be such that all three couplings will evolve together for scales $Q > E_{GUT}$, and one can then speak of a unified coupling.

The running of the gauge couplings is prescribed by a set of first-order non-linear differential equations: the RGEs for the gauge couplings. In general, there is one such equation for each dynamical variable in the theory (*i.e.*, for each gauge coupling, Yukawa coupling, particle and sparticle mass). These equations give the rate of change of each dynamical variable as the scale Q is varied. For the case of a gauge coupling, the rate of change is proportional to (some power of) the gauge coupling itself, and the coefficient of proportionality is called the *beta function.* The beta functions encode the spectrum of the theory, and how the various gauge couplings influence the running of each other (a higher-order effect). Assuming that all supersymmetric particles have a common mass M_{SUSY}, the RGEs (to two-loop order) are:

$$\frac{d\alpha_i^{-1}}{dt} = -\frac{b_i}{2\pi} - \sum_{j=1}^{3} \frac{b_{ij}\alpha_j}{8\pi^2}. \quad (5)$$

where $t = \ln(Q/E_{GUT})$, with Q the running scale and E_{GUT} the unification mass. The one-loop (b_i) and two-loop (b_{ij}) beta functions are given by

$$b_i = \left(\tfrac{33}{5}, 1, -3\right), \quad (6)$$

$$b_{ij} = \begin{pmatrix} \frac{199}{25} & \frac{27}{5} & \frac{88}{5} \\ \frac{9}{5} & 25 & 24 \\ \frac{11}{5} & 9 & 14 \end{pmatrix} . \tag{7}$$

These equations are valid from $Q = M_{SUSY}$ up to $Q = E_{GUT}$. For $M_Z < Q < M_{SUSY}$ an analogous set of equations holds, but with beta functions which reflect the non-supersymmetric nature of the theory (*i.e.*, with all the sparticles decoupled),

$$b'_i = \left(\frac{41}{10}, -\frac{19}{6}, -7\right) , \tag{8}$$

$$b'_{ij} = \begin{pmatrix} \frac{199}{50} & \frac{27}{10} & \frac{44}{5} \\ \frac{9}{10} & \frac{35}{6} & 12 \\ \frac{11}{10} & \frac{9}{2} & -26 \end{pmatrix} . \tag{9}$$

The non-supersymmetric equations are supplemented with the initial conditions given in Eq. (4).

If the above is all the physics which is incorporated in the study of the convergence of the gauge couplings, then it is easy to see that the couplings will always meet at some scale E_{GUT}, provided that M_{SUSY} is tuned appropriately. This is a simple consequence of euclidean geometry, as can be seen from Eq. (5). Neglecting the higher-order terms, we see that as a function of t, α_i^{-1} are just straight lines. In fact, the slope of these lines changes at $Q = M_{SUSY}$, where the beta functions change. The convergence of three straight lines with a change in slope is then guaranteed by euclidean geometry, as long as the point where the slope changes is tuned appropriately. (This fact was pointed out by A. Peterman and one of us in 1979 [3].) On the other hand, the gauge couplings in non-supersymmetric $SU(5)$ do not converge [4]. The precise LEP measurements of the gauge couplings gave new life to the field [5, 6, 7] and produced claims that the convergence of the couplings needed a change in slope at $M_{SUSY} \sim 1\,\text{TeV}$ [8].

This prediction for the likely scale of the supersymmetric spectrum (*i.e.*, $M_{SUSY} \sim 1\,\text{TeV}$ [8]) is in fact *unjustified* [9, 10]. The reason is simple: the physics at the unification scale, which is used to predict the value of M_{SUSY}, has been ignored completely. In fact, such a geometrical picture of convergence of the gauge couplings is physically inconsistent, since for scales $Q > E_{GUT}$ the gauge couplings will depart again, as can be seen in Fig. 1 (of Amaldi *et. al.* [8]). One must consider a unified theory to be assured that the couplings will remain unified, as shown in Fig. 2. This entails the study of a new kind of effect, namely the effects on the running of the gauge couplings produced by the degrees of freedom which are excited near the unification scale (*i.e.*, the *heavy threshold effects*) [10, 11, 12, 13]. In fact, the whole concept of a single unification point needs to be abandoned. The upshot of all this is that the theoretical uncertainties on the values of the parameters describing the heavy GUT particles are such that the above prediction for M_{SUSY} [8] is washed out completely [14, 15]. Furthermore, the insertion of a realistic spectrum of sparticles at low energies (as opposed to an unrealistic common M_{SUSY} mass) plus the calculation of the EGM

Figure 1: In this well publicized example of unification of couplings (Fig. 2 of Ref. [8]), the divergence of the couplings for scales above E_{GUT} (μ and M_{GUT} in the figure) is clear, as is the sharp change in slope of the lines at low energies. Note that neither threshold effects ("light" or "heavy") nor the evolution of the gaugino masses (EGM) are included, and that these are crucial in the determination of where the Superworld could start showing evidence for its existence. Notice that the best fit for the "geometrical" convergence of the couplings predicts M_{SUSY} at 10^3 GeV, and that a low value of $\alpha_3(M_Z) = 0.108$ has been used in this analysis.

Figure 2: The convergence of the gauge couplings $(\alpha_1, \alpha_2, \alpha_3)$ at E_{GUT} is followed by the unification into a unique α_{GUT} above E_{GUT}. The RGEs include the heavy and light thresholds plus the evolution of gaugino masses. These results are obtained using as input the world-average value of $\alpha_3(M_Z)$ and comparing the predictions for $\sin^2\theta(M_Z)$ and $\alpha_{em}^{-1}(M_Z)$ with the experimental results. The χ^2 constructed using these two physically measured quantities allows to get the best E_{GUT}, $\alpha_{GUT}(E_{GUT})$, and $\alpha_3(M_Z)$ (shown). Because all relevant effects have been included, the RGEs can go down to M_Z (*c. f.* Fig. 1). Notice that M_X corresponds to the heavy threshold. The χ^2 definition is based on physical quantities: $\chi^2 = \{[\sin^2\theta(M_Z)]_{exp} - [\sin^2\theta(M_Z)]_{th}\}^2/[\sigma_s]^2 + \{[\alpha_{em}(M_Z)]_{exp} - [\alpha_{em}(M_Z)]_{th}\}^2/[\sigma_e]^2$

Figure 3: This is the best proof that the convergence of the gauge couplings can be obtained with M_{SUSY} at an energy level as low as M_Z. Notice that the effects of "light" and "heavy" thresholds have been accounted for, as well as the Evolution of Gaugino Masses [14,17]. This is Fig. 2 of Ref. [18]. E_{SU} is the string unification scale.

effect (Evolution of Gaugino Masses) blurs the issue even more [11, 14, 16, 17]. *Thus, it is perfectly possible to obtain the unification of the gauge couplings, with supersymmetric particle masses as low as experimentally allowed.* The most complete analysis of a unified theory is shown in Fig. 3. Note the unification of the gauge couplings which continues above E_{GUT}. Notice also that "light" and "heavy" thresholds have been duly accounted for, plus other important effects like the evolution of gaugino masses (EGM) [14] quoted above. This effect has in fact been calculated at two loops [17].

A related point is that LEP data do not uniquely demonstrate that the gauge couplings must unify at a scale $E_{GUT} \sim 10^{16}$ GeV [12]. This is probably the simplest conclusion one could draw. However, this conclusion is easily altered by for example considering all experimental and theoretical uncertainties. In fact, once this is done, the value of E_{GUT} can reach the string unification scale, *i.e.*, $E_{GUT} \sim 10^{18}$ GeV, as shown in Fig. 4.

3 Interconnections between the measured quantities due to Unification

The convergence of the gauge couplings implies that given α_e and α_3, one is able to compute the values of $\sin^2\theta_W$, the unification scale E_{GUT}, and the unified coupling α_U. In lowest-order approximation (*i.e.*, neglecting all GUT thresholds, two-loop effects, and taking $M_{SUSY} = M_Z$) one obtains

$$\ln\frac{E_{GUT}}{M_Z} = \frac{\pi}{10}\left(\frac{1}{\alpha_e} - \frac{8}{3\alpha_3}\right), \tag{10}$$

$$\frac{\alpha_e}{\alpha_U} = \frac{3}{20}\left(1 + \frac{4\alpha_e}{\alpha_3}\right), \tag{11}$$

$$\sin^2\theta_W = 0.2 + \frac{7\alpha_e}{15\alpha_3} \quad . \tag{12}$$

These equations provide a rough approximation to the actual values obtained when all effects are included. Nonetheless, they embody the most important dependences on the input parameters. In Fig. 5 we show the relation between E_{GUT} and α_3 for various values of $\sin^2\theta_W$. One can observe that:

$$\alpha_3 \uparrow \quad \Rightarrow \quad E_{GUT} \uparrow \quad \text{for fixed } \sin^2\theta_W \tag{13}$$

$$\sin^2\theta_W \uparrow \quad \Rightarrow \quad E_{GUT} \uparrow \quad \text{for fixed } \alpha_3 \tag{14}$$

$$\alpha_3 \uparrow \quad \Rightarrow \quad \sin^2\theta_W \downarrow \quad \text{for fixed } E_{GUT} \tag{15}$$

These are the most important systematic correlations, which are not really affected by the neglected effects. These correlations are evident in Eqs. (10–12) and in Fig. 5. In this figure we also show the lower bound on E_{GUT} which follows from the proton decay constraint. Clearly a lower bound on $\alpha_3(M_Z)$ results, which allows the world-average

Figure 4: The dependence of E_{GUT} on $\alpha_3(M_Z)$ and on the ratio of the two crucial heavy-threshold masses, M_V/M_Σ. Here m_0 parametrizes the squark and slepton masses, $m_{1/2}$ the gaugino masses, m_0 and m_4 the Higgs-boson masses, and m_4 the Higgsino mass (m_4 is more commonly denoted by μ). Note that the extreme value for E_{GUT} is above 10^{18} GeV.

Figure 5: The unification scale E_{GUT} versus $\alpha_3(M_Z)$ for various values of $\sin^2\theta_W(M_Z)$ within $\pm 2\sigma$ of the world-average value. Also indicated is the lower bound on E_{GUT} from the lower limit on the proton lifetime.

Figure 6: The unification scale E_{GUT} versus M_{SUSY} for different values of $\alpha_3(M_Z)$ and fixed $\sin^2\theta_W(M_Z)$. Note the anticorrelation between M_{SUSY} and E_{GUT}. The experimental lower bound on M_{SUSY} is shown. The lower bound on E_{GUT} from Fig. 5 is also indicated.

Figure 7: The correlation between all measured quantities, $\alpha_3(M_Z)$, $\sin^2\theta_W(M_Z)$, τ_p, the limits on the lightest detectable supersymmetric particle (here represented by M_{SUSY}) and the unification energy scale E_{GUT}.

value. Another interesting result is the anticorrelation between E_{GUT} and M_{SUSY}. This is shown in Fig. 6, where for fixed $\sin^2\theta_W(M_Z)$ we see that increasing $\alpha_3(M_Z)$ increases E_{GUT} (as already noted in Eq. (13)) and decreases M_{SUSY}. Taking for granted this approach (*i.e.*, all supersymmetric particle masses degenerate at M_{SUSY}) for comparison with the large amount of papers published following this logic, in Fig. 7 we see the narrow band left open once the experimental limits on τ_p and M_{SUSY} are imposed. Figure 7 is a guide to understand the qualitative interconnection between the basic experimentally measured quantities, $\alpha_3(M_Z)$, $\sin^2\theta_W(M_Z)$, τ_p, M_{SUSY} and the theoretically wanted E_{GUT}. The experimental lower bounds on the proton lifetime $(\tau_p)_{exp}$ and on M_{SUSY} produce two opposite bounds (lower and upper, respectively) on the unification energy scale E_{GUT}. Note that, in order to make definite predictions on the lightest detectable supersymmetric particle, a detailed model is needed. In particular, it is necessary to incorporate the evolution of all masses. This has been done in Ref. [16] and an example of spectra is shown in Fig. 8. Let us emphasize that the study of the correlations between the basic quantities, as exemplified in Fig. 7, is interesting but should not be mistaken as example of prediction for the superworld. In particular, the introduction of the quantity M_{SUSY} is really misleading.

Figure 8: A detailed spectrum of SUSY particles showing how misleading is to think of a unique quantity M_{SUSY} in order to describe a SUSY particle spectrum.

4 The origin of M_{SUSY} and why it should be abandoned: masses and spectra are needed

The quantity M_{SUSY} was introduced early on when one-loop RGEs for the gauge couplings were believed to be good enough approximation and threshold effects were altogether neglected. The running of the gauge couplings $\alpha_1^{-1}, \alpha_2^{-1}, \alpha_3^{-1}$ was described by straight lines (see Eqs. 5) whose slopes had to change due to the change in the beta functions from the supersymmetric regime Eqs. (6),(7) to the non-supersymmetric regime Eqs. (8),(9). The change of slope was, for simplicity, described by a single parameter M_{SUSY}, but in no case was M_{SUSY} supposed to represent a physical quantity. It is in fact out of the question that the real spectrum of the supersymmetric particles (an example is shown in Fig. 8) can be degenerate and therefore be represented by a single mass value.

The calculations presented in Ref. [16], attempted to determine the supersymmetric particle spectrum by fitting the various sparticle masses in order to obtain the "best possible" unification picture. These calculations are likely the most extensive analysis of the problem with the simultaneous evolution of seventeen dynamical variables with the radiative effects consistently computed. It is interesting to see how the masses of the superpartners of the various gauge particles change as a function of the experimentally measured quantities $\alpha_3(M_Z), \sin^2\theta_W(M_Z), \alpha_e(M_Z)$. Examples for the $\tilde{\mathrm{W}}$ mass, $m_{\tilde{W}}$, and the gluino mass, $m_{\tilde{g}}$, are shown in Figs. 9 and 10, respectively. We also show in Figs. 11 and 12 how the squark and slepton masses change as a function of the basic quantities quoted above ($\alpha_3(M_Z), \sin^2\theta_W(M_Z), \alpha_e(M_Z)$). Let us emphasize that these are the only experimentally known input parameters in our system of coupled equations [16]. A quantity which will, hopefully, be soon determined is the top-quark mass m_t. Our system of coupled equations allows to determine the $\tilde{\mathrm{W}}$ mass versus m_t. This is shown in Fig. 13. Notice that increasing m_t corresponds to lowering the value of $m_{\tilde{W}}$.

It is interesting to quantitatively see the effect of the three basic quantities ($\alpha_3(M_Z), \sin^2\theta_W(M_Z)$, $\alpha_e(M_Z)$) on the variation of $m_{\tilde{W}}$. This is summarized in Table 1: the dominant effect is $\alpha_3(M_Z)$. It should be noted that these results are given for illustrative purposes: *i.e.*, in order to understand how radiative effects influence the masses of the superparticles when the input measured quantities ($\alpha_3(M_Z), \sin^2\theta_W(M_Z), \alpha_e(M_Z)$) change within their limits of errors.

This program has large inherent uncertainties. As mentioned above, and first of all, because of the fact that we cannot ignore the great uncertainties in the physics at the GUT scale. Furthermore, even if we take the simplest approach and assume that the physics at the GUT scale is represented by a unique threshold, then the introduction of experimental and theoretical uncertainties in the seventeen evolution equations relating gauge couplings and masses, corresponds to a variation (within 2σ) of the sparticle spectra from GeV to PeV, as shown in Fig.14. In fact, so far the only boundary condition imposed is the "best fit" for the unification of the gauge couplings. However, even if this programme would have been successful in predicting

Figure 9: The value of the W-ino mass, $m_{\tilde{W}}$, vs. $\alpha_3(M_Z), \sin^2\theta(M_Z)$, and $(m_4/m_{1/2})^2$. Each curve is the result of our iterative system of seventeen evolution equations. The curve showing the variation of $m_{\tilde{W}}, vs. \alpha_3(M_Z)$, or $\sin^2\theta(M_Z)$, or $(m_4/m_{1/2})^2$ is computed keeping the other parameters fixed at the values indicated by the arrows. For the experimentally measured values, $\alpha_3(M_Z), \sin^2\theta(M_Z)$, the world averages have been chosen. For the ratio of primordial masses the value $(m_0/m_{1/2})^2 = 1$ has been taken. The top mass is kept at $m_t = 125$ GeV. Please note that our analysis is valid only above M_Z. The $m_{\tilde{W}}$ values shown below this limit are for illustrative purposes.

Figure 10: Same as figure 9 when the gluino mass, $m_{\tilde{g}}$, is computed.

Figure 11: Same as figure 9 when the squark mass, $m_{\tilde{q}}$, is computed.

Figure 12: Same as figure 9 when the slepton masses are computed. The left (upper curve) and right (lower curve) slepton masses almost coincide.

Figure 13: The value of the W-ino mass $vs.m_t$ for given values of the other inputs, as indicated. This is the only case $(m_{\tilde{W}})$ where the dependence on m_t is shown. We do not show the result for $m_{\tilde{g}}, m_{\tilde{q}}, m_{\tilde{l}}$.

$$\frac{m_{\tilde{W}}(\alpha_3(M_Z)^{WA}{-}2\sigma)}{m_{\tilde{W}}(\alpha_3(M_Z)^{WA}{+}2\sigma)} \simeq 2\times 10^4$$ *[WA values for the experimental inputs:* $\sin^2\theta(M_Z)$, $\alpha_{em}(M_Z)$*]*
$$\frac{m_{\tilde{W}}(\sin^2\theta(M_Z)^{WA}{-}2\sigma)}{m_{\tilde{W}}(\sin^2\theta(M_Z)^{WA}{+}2\sigma)} = 20 \div 50$$ *[WA values for the experimental inputs:* $\alpha_3(M_Z)$, $\alpha_{em}(M_Z)$*]*
$$\frac{m_{\tilde{W}}(1/\alpha_{em}(M_Z)^{WA}{-}2\sigma)}{m_{\tilde{W}}(1/\alpha_{em}(M_Z)^{WA}{+}2\sigma)} = 1.2 \div 1.3$$ *[WA values for the experimental inputs:* $\sin^2\theta(M_Z)$, $\alpha_3(M_Z)$*]*
Allowed range of variation for primordial SUSY breaking mass ratios: $$(m_0/m_{1/2})^2 = 10^{-2} \div 10^2$$ $$(m_4/m_{1/2})^2 = 10^{-2} \div 10^2$$

Table 1: The variation of the W-ino mass corresponding to $\pm 2\sigma$ variation of the experimental inputs, $\alpha_3(M_Z), \sin^2\theta(M_Z), \alpha_{em}(M_Z)$, with respect to their world average values, and to the variation of $(m_0/m_{1/2})^2$ and $(m_4/m_{1/2})^2$ in the indicated range. The data are derived from our system of seventeen coupled evolution equations. The dominant effect is clearly due to $\alpha_3(M_Z)$. Please note that our equations are valid only above the Z^0 mass. Nevertheless the results in terms of $m_{\tilde{W}}$ ratios are given, for illustrative purposes, even when the $+2\sigma$ limit of the experimental inputs pushes $m_{\tilde{W}}$ below M_Z.

Figure 14: The predicted SUSY mass spectrum for three cases, when the measured quantities ($\alpha_3(M_Z)$, $\sin^2\theta(M_Z)$,$\alpha_{em}(M_Z)$) are taken at their world average values and at $\pm 2\sigma$. The ratios of the primordial parameters are kept equal to one. Note the large range where the SUSY spectra could be on the basis of our best experimental and theoretical knowledge.

the lightest detectable supersymmetric particle near the Fermi scale, this would still be far from satisfying. In fact one would like to know why the supersymmetric spectrum should be the way the fit would require it to be. In other words, the real question is: what determines the values of the sparticle masses? And why should these be below $\sim 1\,\mathrm{TeV}$, so that the gauge hierarchy problem is not re-introduced?

5 The new step forward: Supergravity

In order to answer the question posed at the end of Sec. 4, we must abandon "global" supersymmetry and promote this symmetry of nature to be "local". It is local supersymmetry, *i.e.*, Supergravity which provides the means to compute the masses of the sparticles in a non arbitrary *ad hoc* fashion. In fact, the crucial point is the breaking of local supersymmetry. In Supergravity the breaking occurs in a "hidden sector" of the theory, where "gravitational particles" (those introduced when the supersymmetry was made local) may grow vacuum expectation values (vevs) which break supersymmetry spontaneously in the hidden sector. These vevs are best understood as induced dynamically by the condensation of the supersymmetric partners of the hidden sector particles when the gauge group which describes them becomes strongly interacting at some large scale. The splitting of the particles and their partners would then be generated, and would be of the order of the condensation scale ($\sim 10^{12-16}\,\mathrm{GeV}$). However, such huge mass splittings will be transmitted to the "observable" (the normal) sector of the theory through gravitational interactions, since it is only through these interactions that the two sectors communicate. Gravitational interactions produce dampening in the transmission mechanism in such a way that the splittings in the observable sector are usually much more suppressed than those in the hidden sector, and suitable choices of hidden sectors may yield realistic low-energy supersymmetric spectra. This picture of hidden and observable sectors becomes completely natural in the context of superstrings, where models typically contain both sectors and one can study explicitly the predicted spectrum of supersymmetric particles at low energies.

In a large number of models, the supersymmetric particle masses at the unification scale are also "unified". This situation is called *universal soft-supersymmetry-breaking*, and the masses of all scalar partners (*e.g.*, squarks and sleptons) take the common value of m_0, the gaugino (the partners of the gauge bosons) masses are given by $m_{1/2}$, and there is a third parameter (A) which basically parametrizes the mixing of stop-squark mass eigenstates at low energies. The breaking of the electroweak symmetry is obtained dynamically in the context of these models, through the so-called *radiative electroweak symmetry breaking mechanism*, which involves the top-quark mass in a fundamental way [19, 20]. After all these well motivated theoretical ingredients have been incorporated, the models depend on only four parameters: $m_{1/2}, m_0, A$, and the ratio of the two Higgs vacuum expectations values ($\tan\beta$), plus the top-quark mass (m_t).

In generic Supergravity models the five-dimensional parameter space is constrained by phenomenological requirements, such as sparticle and Higgs-boson masses

not in conflict with present experimental lower bounds, a sufficiently long proton lifetime, a sufficiently old Universe (a cosmological constraint on the amount of dark matter in the Universe today), various indirect constraints from well measured rare processes, etc. Contemporary detailed analyses of supergravity models along these lines have been performed shortly before the LEP era [21], within the last two years [22, 23, 24, 25, 26, 27], and also very recently [28]. In $SU(5) \times U(1)$ Supergravity further string-inspired theoretical constraints can be imposed which give m_0 and A as functions of $m_{1/2}$, and thus reduce the dimension of the parameter space down to just two (plus the top quark mass).

It should be pointed out that a rather interesting situation occurs in the so-called *no-scale* framework [29, 30], where all the scales in the theory are obtained - through radiative corrections - from just one basic scale (*i.e.*, the unification scale or the Planck scale). These models have the unparalleled virtue of a vanishing cosmological constant at the tree-level *even after supersymmetry breaking*, and in their unified versions predict that, at E_{SU}, the universal scalar masses and trilinear couplings vanish (*i.e.*, $m_0(E_{SU}) = A(E_{SU}) = 0$) thus the universal gaugino mass ($m_{1/2}$) is the only seed of supersymmetry breaking. Moreover, this unique mass can be determined in principle by minimizing the vacuum energy at the electroweak scale. The generic result is $m_{1/2} \sim M_Z$ [29, 30], in agreement with theoretical prejudices (*i.e.*, "naturalness" $\equiv$ the radiative corrections of a physical quantity cannot exceed the value of the quantity itself). Finally, it should not be forgotten that *no-scale* Supergravity is the infrared solution of superstring theory [31] and therefore the *no-scale* scenario appears very natural in $SU(5) \times U(1)$ Supergravity.

We now discuss the two simplest Supergravity models. One as representative of Field Theory, the other of String Theory. In fact $SU(5)$ Supergravity is not easily derivable from Superstring theory: no one has succeeded so far. The other is easily derivable from Superstring theory – in fact it has already been done [32, 33] – and this is $SU(5) \times U(1)$ Supergravity.

6 The SU(5) Supergravity Model

The $SU(5)$ supergravity model [34] needs to be specified clearly in order to avoid the common misconception that it is simply the so-called MSSM (Minimal Supersymmetric extension of the Standard Model) with the low-energy gauge couplings meeting at very high energies. Two of its elements are particularly important: (i) it is a supergravity model [35] and as such the soft supersymmetry breaking masses which allow unification are in principle calculable and are assumed to be parametrized in terms of $m_{1/2}, m_0, A$; and (ii) there exist dimension-five proton decay operators [36], which are much larger than the usual dimension-six operators, and require either a tuning of the supersymmetry breaking parameters or a large Higgs triplet mass scale, to obtain a sufficiently long proton lifetime [37, 38, 39, 40, 26, 41, 42, 43].

The $SU(5)$ symmetry is broken down to $SU(3) \times SU(2) \times U(1)$ via a vev of the neutral component of the adjoint **24** of Higgs. The low-energy pair of Higgs

doublets are contained in the $\mathbf{5},\overline{\mathbf{5}}$ Higgs representations. Of the various proposals to split the proton-decay-mediating Higgs triplets from the light Higgs doublets, perhaps the most appealing one is the so-called "missing partner mechanism" [44], whereby a **75** of Higgs breaks the gauge symmetry and the $\mathbf{5},\overline{\mathbf{5}}$ pentaplets are coupled to $\mathbf{50},\overline{\mathbf{50}}$ representations ($\mathbf{50}\cdot\mathbf{75}\cdot\mathbf{5}$, $\overline{\mathbf{50}}\cdot\mathbf{75}\cdot\bar{\mathbf{5}}$). The doublets remain massless, while the triplets acquire $\sim M_U$ masses. (Note: M_U and E_{GUT} are used interchangeably to represent the unification mass scale.) We should remark that this non-minimal symmetry breaking mechanism is *not* the one that is usually considered in studies of high-energy threshold effects in gauge coupling unification, where one usually assumes that it is the **24** which effects the breaking.

6.1 Gauge and Yukawa coupling unification

This problem can be tackled at several levels of sophistication, which entail an increasing number of additional assumptions. The most elementary approach consists of running the one-loop supersymmetric gauge coupling RGEs starting with the precisely measured values of $\alpha_e, \alpha_3, \sin^2\theta_w$ at the scale M_Z and discovering that the three gauge couplings meet at the scale $M_U \sim 10^{16}\,\mathrm{GeV}$ [4], neglecting that above M_U they diverge again. More interesting from the theoretical standpoint is to assume that unification must occur, as is the case in the $SU(5)$ supergravity model, and use this constraint to predict the low-energy value of $\sin^2\theta_w$ and α_e in terms of α_3 [10]. The next level of sophistication consists of increasing the accuracy of the RGEs to two-loop level and parametrize the supersymmetric threshold by a single mass parameter between $\sim M_Z$ and $\sim$ few TeV [5, 7, 8, 45, 9, 10]. More realistically, one specifies the whole light supersymmetric spectrum in detail [6, 11, 15, 23, 41, 14, 16, 2, 46], as well as some subtle and important effects such as the evolution of the gaugino masses (EGM) [14, 15], and the effect of the Yukawa couplings on the two-loop gauge coupling RGEs [47]. A final step of sophistication attempts to model the transition from the $SU(3)\times SU(2)\times U(1)$ theory into the $SU(5)$ theory by means of high-energy threshold effects which depend on the masses of the various GUT fields [11, 15, 13, 41, 10, 12, 2] as well as on coefficients of possible non-renormalizable operators [2, 48]. This last step does away completely with the concept of a single "unification mass". In fact, until this last step is actually accounted for somehow, one is not dealing with a true unified theory since otherwise the gauge couplings would diverge again past the unification scale, *i.e.*, "physics is not euclidean geometry".

It is interesting to note that the original hope that precise knowledge of the low energy gauge couplings would constrain the scale of the low-energy supersymmetric particles, did not bear fruit [13, 14, 15], mainly because of the largely unknown GUT threshold effects. More precisely, the supersymmetric particle masses can lie anywhere up to $\sim$ few TeV provided the parameters of the GUT theory are adjusted accordingly.

Another consequence of the $SU(5)$ symmetry is the relation $\lambda_b(M_U)=\lambda_\tau(M_U)$ which when renormalized down to low energies gives a ratio m_b/m_τ in fairly good agreement with experiment [49]. This problem can also be tackled with improving

degree of sophistication [5, 45, 11, 50, 51, 52, 47, 53, 54] and even postulating some high-energy threshold effects [47, 54]. In practice, the $\lambda_b(M_U) = \lambda_\tau(M_U)$ constraint entails a relationship between m_t and $\tan\beta$, *i.e.*, $\tan\beta = \tan\beta(m_t, m_b, \alpha_3)$, as follows: (i) the values of m_b and m_τ, together with $\tan\beta$ determine the low-energy values of λ_b and λ_τ; (ii) the input value of m_t determines the low-energy value of λ_t; (iii) running these three Yukawa couplings up to the unification scale one discovers the above relation between $\tan\beta$ and m_t if the Yukawa unification constraint is satisfied. In actuality, the dependence on m_b and α_3 is quite important. We note that for arbitrary choices of m_t and $\tan\beta$, one obtains values of m_b typically close to or above 5 GeV, whereas popular belief would like to see values below 4.5 GeV. Strict adherence to this prediction for m_b requires that one be in a rather constrained region of the $(m_t, \tan\beta)$ plane, where $\tan\beta \sim 1$ or $\gtrsim 40$ [15, 51, 47, 54], or that m_t be large (above 180 GeV). We do not impose this stringent constraint on the parameter space, hoping that further contributions to the quark masses (as required in $SU(5)$ GUTs to fit the lighter generations also [55]) will relax it somehow.

As noted above, the parameter space of this model can be described in terms of five parameters: $m_t, \tan\beta, m_{1/2}, m_0, A$. In Ref. [42] we performed an exploration of the following hypercube of the parameter space: $\mu > 0, \mu < 0$, $\tan\beta = 2 - 10\,(2)$, $m_t = 100 - 160\,(5)$, $\xi_0 \equiv m_0/m_{1/2} = 0 - 10\,(1)$, $\xi_A \equiv m_A/m_{1/2} = -\xi_0, 0, +\xi_0$, and $m_{1/2} = 50 - 300\,(6)$, where the numbers in parenthesis represent the size of the step taken in that particular direction. (Points outside these ranges have little (a posteriori) likelihood of being acceptable.) Of these $92,235 \times 2 = 184,470$ points, $\approx 25\%$ passed all the standard constraints, *i.e.*, radiative electroweak symmetry breaking and all low-energy phenomenology as described in Ref. [27]. The most important constraint on this parameter space is proton decay, as discussed below. First we discuss some aspects of the gauge coupling unification calculation.

As a first step we used one-loop gauge coupling RGEs and a common supersymmetric threshold at M_Z, to determine M_U, α_U, and $\sin^2\theta_w$, once $\alpha_3(M_Z) = 0.113, 0.120$ and $\alpha_e^{-1}(M_Z) = 127.9$ were given. In Ref. [43] we refined our study including several important features: (i) recalculation of M_U using two-loop gauge coupling RGEs including light supersymmetric thresholds, (ii) exploration of values of α_3 throughout its $\pm1\sigma$ allowed range, and (iii) exploration of low values of $\tan\beta\,(< 2)$ (which maximize the proton lifetime). We used the analytical approximations to the solution of the two-loop gauge coupling RGEs in Ref. [14] to obtain $M_U, \alpha_U, \sin^2\theta_w$. The supersymmetric threshold was treated in great detail [14] with all the sparticle masses obtained from our procedure [42]. Since the sparticle masses vary as one explores the parameter space, one obtains *ranges* for the calculated values. In Table 2 we show the one-loop value for M_U ($M_U^{(0)}$), the two-loop plus supersymmetric threshold corrected unification mass range ($M_U^{(1)}$) [as expected [14] M_U is reduced by both effects], the ratio of the two, and the calculated range of $\sin^2\theta_w$.[1] Note that for $\alpha_3 = 0.118$ (and lower), $\sin^2\theta_w$ is outside the experimental $\pm1\sigma$ range

[1] We should note that these ranges are obtained after all constraints discussed below have been satisfied, the proton decay being the most important one.

Table 2: The value of the one-loop unification mass $M_U^{(0)}$, the two-loop and supersymmetric threshold corrected unification mass range $M_U^{(1)}$, the ratio of the two, and the range of the calculated $\sin^2\theta_w$, for the indicated values of α_3 (the superscript $+\,(-)$ denotes $\mu > 0\,(< 0)$) and $\alpha_e^{-1} = 127.9$. The $\sin^2\theta_w$ values should be compared with the current experimental $\pm 1\sigma$ range $\sin^2\theta_w = 0.2324 \pm 0.0006$ [2]. Lower values of α_3 drive $\sin^2\theta_w$ to values even higher than for $\alpha_3 = 0.118$. All masses in units of 10^{16} GeV.

	$\alpha_3 = 0.126^+$	$\alpha_3 = 0.126^-$	$\alpha_3 = 0.118^+$	$\alpha_3 = 0.118^-$
$M_U^{(0)}$	3.33	3.33	2.12	2.12
$M_U^{(1)}$	$1.60 - 2.13$	$1.60 - 2.05$	$1.02 - 1.35$	$1.02 - 1.30$
$M_U^{(1)}/M_U^{(0)}$	$0.48 - 0.64$	$0.48 - 0.61$	$0.48 - 0.64$	$0.48 - 0.61$
$\sin^2\theta_w$	$0.2315 - 0.2332$	$0.2313 - 0.2326$	$0.2335 - 0.2351$	$0.2332 - 0.2345$

($\sin^2\theta_w = 0.2324 \pm 0.0006$ [2]), whereas $\alpha_3 = 0.126$ gives quite acceptable values.

We do not specify the details of the GUT thresholds and in practice take two of the GUT mass parameters (the masses of the X, Y gauge bosons M_V, and the mass of the adjoint Higgs multiplet M_Σ) to be degenerate with M_U. Since below we allow $M_H < 3M_U$, Table 2 indicates that in our calculations $M_H < 6.4 \times 10^{16}$ GeV. In Ref. [41] it is argued that a more proper upper bound is $M_H < 2M_V$, but M_V cannot be calculated directly, only $(M_V^2 M_\Sigma)^{1/3} < 3.3 \times 10^{16}$ GeV is known from low-energy data [41]. If we take $M_\Sigma = M_V$, this would give $M_H < 2M_V < 6.6 \times 10^{16}$ GeV, which agrees with our present requirement. Below we comment on the case $M_\Sigma < M_V$.

6.2 Proton decay

In the $SU(5)$ supergravity model only the dimension-five–mediated proton decay operators are constraining. In calculating the proton lifetime we consider the typically dominant decay modes $p \to \bar{\nu}_{\mu,\tau} K^+$ and neglect all other possible modes. Schematically the lifetime is given by[2]

$$\tau_p \equiv \tau(p \to \bar{\nu}_{\mu,\tau} K^+) \sim \left| M_H \sin 2\beta \frac{1}{f} \frac{1}{1 + y^{tK}} \right|^2 . \tag{16}$$

Here M_H is the mass of the exchanged GUT Higgs triplet which on perturbative grounds is assumed to be bounded above by $M_H < 3M_U$ [3] [38, 26, 41]; $\sin 2\beta = 2\tan\beta/(1 + \tan^2\beta)$, thus τ_p "likes" small $\tan\beta$ (we find that only $\tan\beta \lesssim 6$ is

[2]Throughout our calculations we have used the explicit proton decay formulas in Ref. [39].

[3]This relation assumes implicitly that all the components of the **24** superfield are nearly degenerate in mass [41].

allowed); y^{tK} represents the calculable ratio of the third- to the second-generation contributions to the dressing one-loop diagrams. An unkown phase appears in this ratio (which has generally $|y^{tK}| \ll 1$) and we always consider the weakest possible case of destructive interference. Finally f represents the sparticle-mass–dependent dressing one-loop function which decreases asymptotically with large sparticle masses.

In Fig. 15 (top row) we show a scatter plot of $(\tau_p, m_{\tilde{g}})$. The various 'branches' correspond to fixed values of ξ_0. Note that for $\xi_0 < 3$, $\tau_p < \tau_p^{exp} = 1 \times 10^{32}\,\mathrm{y}$ (at 90% C.L. [56]). Also, for a given value of ξ_0, there is a corresponding allowed interval in $m_{\tilde{g}}$. The lower end of this interval is determined by the fact that $\tau_p \propto 1/f^2$, and $f \approx m_{\chi_1^+}/m^2_{\tilde{q}} \propto 1/m_{\tilde{g}}(c+\xi_0^2)$, in the proton-decay–favored limit of $\mu \gg M_W$; thus $m_{\tilde{g}}(c+\xi_0^2) >$ constant. The upper end of the interval follows from the requirement $m_{\tilde{q}}(\propto m_{\tilde{g}}\sqrt{c+\xi_0^2}) < 1\,\mathrm{TeV}$. Statistically speaking, the proton decay cut is quite severe, allowing only about $\sim 1/10$ of the points which passed all the standard constraints, independently of the sign of μ.

Note that if we take $M_H = M_U$ (instead of $M_H = 3M_U$), then $\tau_p \longrightarrow \frac{1}{9}\tau_p$ and all points in Fig. 15 would become excluded. To obtain a rigorous lower bound on M_H, we would need to explore the lowest possible allowed values of $\tan\beta$ (in Fig. 15, $\tan\beta \geq 2$). Roughly, since the dominant $\tan\beta$ dependence of τ_p is through the explicit $\sin 2\beta$ factor, the upper bound $\tau_p \lesssim 8 \times 10^{32}\,\mathrm{y}$ for $\tan\beta = 2$, would become $\tau_p \lesssim 1 \times 10^{33}\,\mathrm{y}$ for $\tan\beta = 1$. Therefore, the current experimental lower bound on τ_p would imply $M_H \gtrsim M_U$. Note also that SuperKamiokande ($\tau_p^{exp} \approx 2 \times 10^{33}\,\mathrm{y}$) should be able to probe the whole allowed range of τ_p values.

The actual value of $\alpha_3(M_Z)$ used in the calculations ($\alpha_3 = 0.120$ in Fig. 15) has a non-negligible effect on some of the final results, mostly due to its effect on the value of $M_H = 3M_U$: larger values of α_3 increase M_U and therefore τ_p, and thus open up the parameter space, and viceversa. For example, for $\alpha_3 = 0.113\,(0.120)$ we get $m_{\tilde{g}} \lesssim 550\,(800)\,\mathrm{GeV}$, $\xi_0 \geq 5\,(3)$, and $\tau_p \lesssim 4\,(8) \times 10^{32}\,\mathrm{y}$.

More details on the matter instability problem are given in Sec. 11.1, where we present detailed calculations for the underground labs.

6.3 Neutralino relic density

The study of the relic density of neutralinos requires the knowledge of the total annihilation amplitude $\chi\chi \to all$. The latter depends on the model parameters to determine all masses and couplings. Previously [57, 58] we have advocated the study of this problem in the context of supergravity models with radiative electroweak symmetry breaking, since then only a few parameters (five or less) are needed to specify the model completely. In particular, one can explore the whole parameter space and draw conclusions about a complete class of models. The ensuing relationships among the various masses and couplings have been found to yield results which depart from the conventional minimal supersymmetric standard model (MSSM) lore, where no such relations exist. In the $SU(5)$ supergravity model we have just shown that its five-dimensional parameter space is strongly constrained by the proton lifetime. It

Figure 15: Scatter plot of the proton lifetime $\tau_p \equiv \tau(p \to \bar{\nu}_{\mu,\tau} K^+)$ versus the gluino mass for the hypercube of the parameter space explored. The unification mass is calculated in one-loop approximation assuming a common supersymmetric threshold at M_Z, and $M_H = 3M_U$ is assumed. The current experimental lower bound is $\tau_p^{exp} = 1 \times 10^{32}\,$y. The various 'branches' correspond to fixed values of ξ_0 as indicated (the labelling applies to all four windows). The bottom row includes the cosmological constraint. The upper bound on $m_{\tilde{g}}$ follows from the requirement $m_{\tilde{q}} < 1\,$TeV.

was first noticed in Ref. [59] that the neutralino relic density for the proton-

decay allowed points in parameter space is large, *i.e.*, $\Omega_\chi h_0^2 \gg 1$, and therefore in conflict with current cosmological expectations: requiring that the Universe be older than the oldest known stars implies $\Omega_0 h_0^2 \leq 1$ [60].

In Refs. [59, 42, 43] the neutralino relic density has been computed following the methods of Refs. [57, 58]. In Fig. 15 (bottom row) we show the effect of the cosmological constraint on the parameter space allowed by proton decay. Only $\sim 1/6$ of the points satisfy $\Omega_\chi h_0^2 \leq 1$. This result is not unexpected since proton decay is suppressed by heavy sparticle masses, whereas $\Omega_\chi h_0^2$ is enhanced. Therefore, a delicate balance needs to be attained to satisfy both constraints simultaneously. Note that the subset of cosmologically allowed points does not change the range of possible τ_p values, although it depletes the constant-ξ_0 'branches'.

The effect of the cosmological constraint is perhaps more manifest when one considers the correlation between m_h and $m_{\chi_1^\pm}$ after imposing the (*e.g.*, *weaker*) proton decay constraint, but with and without imposing the cosmological constraint. This contrast is shown in Fig. 16.

The main conclusion is that the relic density can be small only near the h- and Z-pole resonances, *i.e.*, for $m_\chi \approx \frac{1}{2} m_{h,Z}$ [42, 61], since in this case the annihilation cross section is enhanced. It is important to note that in this type of calculations the thermal average of the annihilation cross section is usually computed using an expansion around threshold (*i.e.*, $\sqrt{s} = 2m_\chi$) [62]. In Ref. [63] however, it has been pointed out that the resulting thermal average can be quite inaccurate near poles and thresholds of the annihilation cross section, which is precisely the case for the points of interest in the $SU(5)$ model. In Ref. [61, 64] the relic density calculation has been redone following the more accurate methods of Ref. [63]. The result is that the poles are broader and shallower, and thus the cosmological constraint is weakened with respect to the standard (using the expansion) procedure of performing the thermal average. However, qualitatively the cosmologically allowed region of parameter space is not changed. This result is shown in Fig. 17, where the points in parameter space allowed by the stricter proton decay constraint and cosmology are shown in the $(m_{\chi_1^\pm}, m_h)$ plane. Note that the "exact" calculation of the relic density allows more points in parameter space, therefore the plot is more populated in this case.

6.4 Mass ranges and relations

Since the proton decay constraint generally requires $|\mu| \gg M_W$ (and to a somewhat lesser extent also $|\mu| \gg M_2$), the lightest chargino will have mass $m_{\chi_1^+} \approx M_2 \approx 0.3 m_{\tilde{g}}$, whereas the two lightest neutralinos will have masses $m_\chi \approx M_1 \approx \frac{1}{2} M_2$ and $m_{\chi_2^0} \approx M_2$ [26, 65]. Inclusion of the cosmological constraint does not affect significantly the range of sparticle masses in Sec. 6.2. The value of α_3 does not affect these mass relations either, although the particle mass ranges do change

$$m_\chi < 85\,(115)\,\mathrm{GeV}, \qquad m_{\chi_2^0,\chi_1^+} < 165\,(225)\,\mathrm{GeV}, \qquad \text{for } \alpha_3 = 0.113\,(0.120). \quad (17)$$

Figure 16: The allowed region in parameter space which satisfies the *weaker* proton decay constraint, before and after the imposition of the cosmological constraint. Note that when the cosmological constraint is imposed (bottom row), an interesting correlation between the two particle masses arises.

The reason is simple: higher values of α_3 increase M_U and therefore $M_H\,(=3M_U)$, which in turn weakens the proton decay constraint. We also find that the one-loop corrected lightest Higgs boson mass (m_h) is bounded above by

$$m_h \lesssim 110\,(100)\,\mathrm{GeV}, \tag{18}$$

independently of the sign of μ, the value of α_3, or the cosmological constraint; the stronger bound holds when the stricter proton decay constraint is enforced. In Fig. 17 we have shown m_h versus $m_{\chi_1^+}$ for $\tan\beta = 1.5, 1.75, 2$; for the maximum allowed $\tan\beta$ value (≈ 3.5), $m_h \lesssim 100\,\mathrm{GeV}$. Note that for $\mu > 0$, $m_h \approx 50\,\mathrm{GeV}$, and $m_{\chi_1^\pm} \gtrsim 100\,\mathrm{GeV}$, there is a sparsely populated area with highly fine-tuned points in parameter space ($m_t \approx 100\,\mathrm{GeV}$, $\tan\beta \approx 1.5$, $\xi_A \equiv A/m_{1/2} \approx \xi_0 \approx 6$). This figure shows an experimentally interesting correlation when the cosmological constraints are imposed,

$$m_h \gtrsim 72\,\mathrm{GeV} \Rightarrow m_{\chi_1^+} \lesssim 100\,\mathrm{GeV}. \tag{19}$$

Figure 17: The points in parameter space of the $SU(5)$ supergravity model which satisfy the stricter proton decay constraint and the cosmological constraint with the relic density computed in approximate and accurate way. Note that since the "exact" calculation of the relic density allows more points in parameter space, the plot is more populated in this case.

The bands of points towards low values of m_h represent the discrete choices of $\tan\beta = 1.5, 1.75$. The voids between these bands are to be understood as filled by points with $1.5 \lesssim \tan\beta \lesssim 1.75$. For $m_{\chi_1^\pm} > 106\,(92)$ GeV (for $\mu > 0\,(\mu < 0)$), we obtain $m_h \lesssim 50\,(56)$ GeV and Higgs detection at LEP should be immediate. The correlations among the lightest chargino and neutralino masses imply analogous results for $(m_h, m_{\chi_2^0})$ and (m_h, m_χ),

$$m_h \gtrsim 80\,\mathrm{GeV} \Rightarrow m_{\chi_2^0} \lesssim 90\,(110)\,\mathrm{GeV}, \quad m_\chi \lesssim 48\,(60)\,\mathrm{GeV}, \tag{20}$$

for $\alpha_3 = 0.113\,(0.120)$. These correlations can be understood in the following way: since we find that $m_A \gg M_Z$, then $m_h \approx |\cos 2\beta| M_Z +$(rad. corr.). In the situation we consider here, we have determined that all of the allowed points for $m_{\tilde{g}} > 400$ GeV correspond to $\tan\beta = 2$. This implies that the tree-level contribution to m_h is ≈ 55 GeV. We also find that the cosmology cut restricts $m_t < 130(140)$ GeV for $\mu > 0 (\mu < 0)$ in this range of $m_{\tilde{g}}$. Therefore, the radiative correction contribution to m_h^2 ($\propto m_t^4$) will be modest in this range of $m_{\tilde{g}}$. This explains the depletion of points for $m_h \gtrsim 80$ GeV in Fig. 17 and leads to the mass relationships in Eqs. (19,20).

In this model the only light particles are the lightest Higgs boson ($m_h \lesssim 100$ GeV), the two lightest neutralinos ($m_{\chi_1^0} \approx \frac{1}{2} m_{\chi_2^0} \lesssim 75$ GeV), and the lightest chargino ($m_{\chi_1^\pm} \approx m_{\chi_2^0} \lesssim 150$ GeV). The gluino and the lightest stop can be light ($m_{\tilde{g}} \approx 160 - 460$ GeV, $m_{\tilde{t}_1} \approx 170 - 825$ GeV), but for most of the parameter space are not within the reach of Fermilab.

In Ref. [66] it has been shown that the actual LEP lower bound on the lightest Higgs boson mass is improved in the class of supergravity models with radiative electroweak symmetry breaking which we consider here, one gets $m_h \gtrsim 60$ GeV. In Sec. 9.1 below we discuss the details of this procedure. For now it suffices to note that the improved bound $m_h \gtrsim 60$ GeV mostly restricts low values of $\tan\beta$ and therefore the $SU(5)$ supergravity model where $\tan\beta \lesssim 3.5$ [43]. Above we obtained upper bounds on the light particle masses in this model ($\tilde{g}, h, \chi_{1,2}^0, \chi_1^\pm$) for $m_h > 43$ GeV. In particular, it was found that $m_{\chi_1^\pm} \gtrsim 100$ GeV was only possible for $m_h \lesssim 50$ GeV. The improved bound on m_h immediately implies the following considerably stronger upper bounds

$$m_{\chi_1^0} \lesssim 52(50)\,\mathrm{GeV}, \tag{21}$$
$$m_{\chi_2^0} \lesssim 103(94)\,\mathrm{GeV}, \tag{22}$$
$$m_{\chi_1^\pm} \lesssim 104(92)\,\mathrm{GeV}, \tag{23}$$
$$m_{\tilde{g}} \lesssim 320\,(405)\,\mathrm{GeV}, \tag{24}$$

for $\mu > 0$ ($\mu < 0$).

A related consequence is that the mass relation $m_{\chi_2^0} > m_{\chi_1^0} + m_h$ is not satisfied for any of the remaining points in parameter space and therefore the $\chi_2^0 \to \chi_1^0 h$ decay mode is not kinematically allowed. Points where such mode was previously allowed led to a vanishing trilepton signal in the reaction $p\bar{p} \to \chi_1^\pm \chi_2^0$ at Fermilab (thus the

name 'spoiler mode') [67]. The improved situation now implies at least one event per $100\,\mathrm{pb}^{-1}$ for all remaining points in parameter space (see Sec. 8).

7 SU(5)xU(1) Supergravity

7.1 General features

The model we consider [68] is a generalization of that presented in Ref. [69], and contains the following $SU(5)\times U(1)$ fields:

1. three generations of quark and lepton fields $F_i, \bar{f}_i, l_i^c$, $i=1,2,3$;
2. two pairs of Higgs $\mathbf{10}$,$\overline{\mathbf{10}}$ representations $H_i, \bar{H}_i$, $i=1,2$;
3. one pair of "electroweak" Higgs $\mathbf{5}$,$\overline{\mathbf{5}}$ representations $h, \bar{h}$;
4. three singlet fields $\phi_{1,2,3}$.

Under $SU(3)\times SU(2)$ the various $SU(5)\times U(1)$ fields decompose as follows:

$$\begin{aligned} F_i &= \{Q_i, d_i^c, \nu_i^c\}, \quad \bar{f}_i=\{L_i,u_i^c\}, \quad l_i^c=e_i^c, && (25)\\ H_i &= \{Q_{H_i}, d_{H_i}^c, \nu_{H_i}^c\}, \quad \bar{H}_i=\{Q_{\bar{H}_i}, d_{\bar{H}_i}^c, \nu_{\bar{H}_i}^c\}, && (26)\\ h &= \{H, D\}, \quad \bar{h}=\{\bar{H},\bar{D}\}. && (27)\end{aligned}$$

The most general effective[4] superpotential consistent with $SU(5)\times U(1)$ symmetry is given by

$$\begin{aligned} W &= \lambda_1^{ij}F_iF_jh+\lambda_2^{ij}F_i\bar{f}_j\bar{h}+\lambda_3^{ij}\bar{f}_il_j^ch+\mu h\bar{h}+\lambda_4^{ij}H_iH_jh+\lambda_5^{ij}\bar{H}_i\bar{H}_j\bar{h}\\ &+ \lambda_{1'}^{ij}H_iF_jh+\lambda_{2'}^{ij}H_i\bar{f}_j\bar{h}+\lambda_6^{ijk}F_i\bar{H}_j\phi_k+w^{ij}H_i\bar{H}_j+\mu^{ij}\phi_i\phi_j. \qquad (28)\end{aligned}$$

Symmetry breaking is effected by non-zero vevs $\left\langle \nu_{H_i}^c\right\rangle = V_i$, $\left\langle \nu_{\bar{H}_i}^c\right\rangle=\bar{V}_i$, such that $V_1^2+V_2^2=\bar{V}_1^2+\bar{V}_2^2$.

7.1.1 Higgs doublet and triplet mass matrices

The Higgs doublet mass matrix receives contributions from $\mu h\bar{h}\to\mu H\bar{H}$ and $\lambda_{2'}^{ij}H_i\bar{f}_j\bar{h}$ $\to\lambda_{2'}^{ij}V_iL_j\bar{H}$. The resulting matrix is

$$\mathcal{M}_2 = \begin{array}{c c} & \bar{H} \\ \begin{array}{c} H\\ L_1\\ L_2\\ L_3\end{array} & \begin{pmatrix} \mu \\ \lambda_{2'}^{i1}V_i \\ \lambda_{2'}^{i2}V_i \\ \lambda_{2'}^{i3}V_i \end{pmatrix} \end{array}. \qquad (29)$$

[4] To be understood in the string context as arising from cubic and higher order terms [70, 71].

To avoid fine-tunings of the $\lambda_{2'}^{ij}$ couplings we must demand $\lambda_{2'}^{ij} \equiv 0$, so that $\bar{H}$ remains light.

The Higgs triplet matrix receives several contributions: $\mu h\bar{h} \to \mu D\bar{D}$; $\lambda_{1'}^{ij} H_i F_j h \to \lambda_{1'}^{ij} V_i d_j^c D$; $\lambda_4^{ij} H_i H_j h \to \lambda_4^{ij} V_i d_{H_j}^c D$; $\lambda_5^{ij} \bar{H}_i \bar{H}_j \bar{h} \to \lambda_5^{ij} \bar{V}_i d_{\bar{H}_j}^c \bar{D}$; $w^{ij} d_{H_i}^c d_{\bar{H}_j}^c$. The resulting matrix is[5]

$$\mathcal{M}_3 = \begin{array}{c} \\ D \\ d^c_{\bar{H}_1} \\ d^c_{\bar{H}_2} \end{array} \!\!\begin{array}{c} \begin{array}{cccccc} \bar{D} & d^c_{H_1} & d^c_{H_2} & d^c_1 & d^c_2 & d^c_3 \end{array} \\ \begin{pmatrix} \mu & \lambda_4^{i1} V_i & \lambda_4^{i2} V_i & \lambda_{1'}^{i1} V_i & \lambda_{1'}^{i2} V_i & \lambda_{1'}^{i3} V_i \\ \lambda_5^{i1} \bar{V}_i & w_{11} & w_{12} & 0 & 0 & 0 \\ \lambda_5^{i2} \bar{V}_i & w_{21} & w_{22} & 0 & 0 & 0 \end{pmatrix} \end{array} . \tag{30}$$

Clearly three linear combinations of $\{\bar{D}, d^c_{H_{1,2}}, d^c_{1,2,3}\}$ will remain light. In fact, such a general situation will induce a mixing in the down-type Yukawa matrix $\lambda_1^{ij} F_i F_j h \to \lambda_1^{ij} Q_i d_j^c H$, since the d_j^c will need to be re-expressed in terms of these mixed light eigenstates.[6] This low-energy quark-mixing mechanism is an explicit realization of the general extra-vector-abeyance (EVA) mechanism of Ref. [72]. As a first approximation though, in what follows we will set $\lambda_{1'}^{ij} = 0$, so that the light eigenstates are $d^c_{1,2,3}$.

7.1.2 Neutrino see-saw matrix

The see-saw neutrino matrix receives contributions from: $\lambda_2^{ij} F_i \bar{f}_j \bar{h} \to m_u^{ij} \nu_i^c \nu_j$; $\lambda_6^{ijk} F_i \bar{H}_j \phi_k \to \lambda_6^{ijk} \bar{V}_j \nu_i^c \phi_k$; $\mu^{ij} \phi_i \phi_j$. The resulting matrix is[7]

$$\mathcal{M}_\nu = \begin{array}{c} \\ \nu_i \\ \nu_i^c \\ \phi_i \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \nu_j & \nu_j^c & \phi_j \end{array} \\ \begin{pmatrix} 0 & m_u^{ji} & 0 \\ m_u^{ij} & 0 & \lambda_6^{ikj} \bar{V}_k \\ 0 & \lambda_6^{jki} \bar{V}_k & \mu^{ij} \end{pmatrix} \end{array} . \tag{31}$$

7.1.3 Numerical scenario

To simplify the discussion we will assume, besides[8] $\lambda_{1'}^{ij} = \lambda_{2'}^{ij} \equiv 0$, that

$$\lambda_4^{ij} = \delta^{ij} \lambda_4^{(i)}, \quad \lambda_5^{ij} = \delta^{ij} \lambda_5^{(i)}, \quad \lambda_6^{ijk} = \delta^{ij} \delta^{ik} \lambda_6^{(i)}, \tag{32}$$

$$\mu^{ij} = \delta^{ij} \mu_i, \quad w^{ij} = \delta^{ij} w_i. \tag{33}$$

[5]The zero entries in $\mathcal{M}_3$ result from the assumption $\langle \phi_k \rangle = 0$ in $\lambda_6^{ijk} F_i \bar{H}_j \phi_k$.

[6]Note that this mixing is on top of any structure that λ_1^{ij} may have, and is the only source of mixing in the typical string model-building case of a diagonal λ_2 matrix.

[7]We neglect a possible higher-order contribution which could produce a non-vanishing $\nu_i^c \nu_j^c$ entry [73].

[8]In Ref. [69] the discrete symmetry $H_1 \to -H_1$ was imposed so that these couplings automatically vanish when $H_2, \bar{H}_2$ are not present. This symmetry (generalized to $H_i \to -H_i$) is not needed here since it would imply $w^{ij} \equiv 0$, which is shown below to be disastrous for gauge coupling unification.

These choices are likely to be realized in string versions of this model and will not alter our conclusions below. In this case the Higgs triplet mass matrix reduces to

$$\mathcal{M}_3 = \begin{array}{c} \\ D \\ d^c_{\bar{H}_1} \\ d^c_{\bar{H}_2} \end{array} \begin{array}{c} \begin{array}{ccc} \bar{D} & d^c_{H_1} & d^c_{H_2} \end{array} \\ \begin{pmatrix} \mu & \lambda_4^{(1)} V_1 & \lambda_4^{(2)} V_2 \\ \lambda_5^{(1)} \bar{V}_1 & w_1 & 0 \\ \lambda_5^{(2)} \bar{V}_2 & 0 & w_2 \end{pmatrix} \end{array} . \tag{34}$$

Regarding the $(3,2)$ states, the scalars get either eaten by the X, Y $SU(5)$ heavy gauge bosons or become heavy Higgs bosons, whereas the fermions interact with the $\widetilde{X}, \widetilde{Y}$ gauginos through the following mass matrix [33]

$$\mathcal{M}_{(3,2)} = \begin{array}{c} \\ Q_{H_1} \\ Q_{H_2} \\ \widetilde{X} \end{array} \begin{array}{c} \begin{array}{ccc} Q_{\bar{H}_1} & Q_{\bar{H}_2} & \widetilde{Y} \end{array} \\ \begin{pmatrix} w_1 & 0 & g_5 V_1 \\ 0 & w_2 & g_5 V_2 \\ g_5 \bar{V}_1 & g_5 \bar{V}_2 & 0 \end{pmatrix} \end{array} . \tag{35}$$

The lightest eigenvalues of these two matrices (denoted generally by d^c_H and Q_H respectively) constitute the new relatively light particles in the spectrum, which are hereafter referred to as the "*gap*" particles since with suitable masses they bridge the gap between unification masses at 10^{16} GeV and 10^{18} GeV.

Guided by the phenomenological requirement on the gap particle masses, *i.e.*, $M_{Q_H} \gg M_{d^c_H}$ [74], we consider the following explicit numerical scenario

$$\lambda_4^{(2)} = \lambda_5^{(2)} = 0, \quad V_1, \bar{V}_1, V_2, \bar{V}_2 \sim V \gg w_1 \gg w_2 \gg \mu, \tag{36}$$

which would need to be reproduced in a viable string-derived model. From Eq. (34) we then get $M_{d^c_{H_2}} = M_{d^c_{\bar{H}_2}} = w_2$, and all other mass eigenstates $\sim V$. Furthermore, $\mathcal{M}_{(3,2)}$ has a characteristic polynomial $\lambda^3 - \lambda^2(w_1 + w_2) - \lambda(2V^2 - w_1 w_2) + (w_1 + w_2)V^2 = 0$, which has two roots of $\mathcal{O}(V)$ and one root of $\mathcal{O}(w_1)$. The latter corresponds to $\sim (Q_{H_1} - Q_{H_2})$ and $\sim (Q_{\bar{H}_1} - Q_{\bar{H}_2})$. In sum then, the gap particles have masses $M_{Q_H} \sim w_1$ and $M_{d^c_H} \sim w_2$, whereas all other heavy particles have masses $\sim V$.

The see-saw matrix reduces to

$$\mathcal{M}_\nu = \begin{array}{c} \\ \nu_i \\ \nu^c_i \\ \phi_i \end{array} \begin{array}{c} \begin{array}{ccc} \nu_i & \nu^c_i & \phi_i \end{array} \\ \begin{pmatrix} 0 & m^i_u & 0 \\ m^i_u & 0 & \lambda^{(i)} \bar{V}_i \\ 0 & \lambda^{(i)} \bar{V}_i & \mu^i \end{pmatrix} \end{array} , \tag{37}$$

for each generation. The physics of this see-saw matrix has been discussed in Ref. [73] and more generally in Ref. [75], where it was shown to lead to an interesting amount of hot dark matter (ν_τ) and an MSW-effect (ν_e, ν_μ) compatible with all solar neutrino data. Moreover, the out-of-equilibrium decays of the ν^c "flipped neutrino" fields in the early Universe induce a lepton number asymmetry which is later processed into a baryon number asymmetry by non-perturbative electroweak processes [76, 75]. All these phenomena can occur in the same region of parameter space.

7.1.4 Proton decay

The dimension-six operators mediating proton decay in this model are highly suppressed due to the large mass of the X, Y gauge bosons ($\sim M_U = 10^{18}$ GeV). Higgsino mediated dimension-five operators exist and are naturally suppressed in the model of Ref. [69]. The reason for this is that the Higgs triplet mixing term $\mu h\bar{h} \to \mu D\bar{D}$ is small ($\mu \sim M_Z$), whereas the Higgs triplet mass eigenstates obtained from Eq. (30) by just keeping the 2×2 submatrix in the upper left-hand corner, are always very heavy ($\sim V$). The dimension-five mediated operators are then proportional to μ/V^2 and thus the rate is suppressed by a factor of $(\mu/V)^2 \ll 1$ relative to the unsuppressed case found in the standard $SU(5)$ model.

In the model presented here, the Higgs triplet mixing term is still $\mu D\bar{D}$. However, the exchanged mass eigenstates are not necessarily all very heavy. In fact, above we have demanded the existence of a relatively light ($\sim w_1$) Higgs triplet state (d^c_H). In this case the operators are proportional to $\mu\alpha_i\bar{\alpha}_i/\mathcal{M}_i^2$, where $\mathcal{M}_i$ is the mass of the i-th exchanged eigenstate and $\alpha_i, \bar{\alpha}_i$ are its $D, \bar{D}$ admixtures. In the scenario described above, the relatively light eigenstates ($d^c_{H_2}, d^c_{\bar{H}_2}$) contain no $D, \bar{D}$ admixtures, and the operator will again be $\propto \mu/V^2$.

Note however that if conditions (36) (or some analogous suitability requirement) are not satisfied, then diagonalization of $\mathcal{M}_3$ in Eq. (34) may re-introduce a sizeable dimension-five mediated proton decay rate, depending on the value of the $\alpha_i, \bar{\alpha}_i$ coefficients. To be safe one should demand [26, 41, 43]

$$\frac{\mu\alpha_i\bar{\alpha}_i}{\mathcal{M}_i^2} \lesssim \frac{1}{10^{17}\,\mathrm{GeV}}. \tag{38}$$

For the higher values of $M_{d^c_H}$ in Table 3 (see below), this constraint can be satisfied for not necessarily small values of $\alpha_i, \bar{\alpha}_i$.

7.1.5 Gauge coupling unification

Since we have chosen $V \sim M_U = M_{SU} = 10^{18}$ GeV, this means that the Standard Model gauge couplings should unify at the scale M_U. However, their running will be modified due to the presence of the gap particles. Note that the underlying $SU(5) \times U(1)$ symmetry, even though not evident in this respect, is nevertheless essential in the above discussion. The masses M_Q and $M_{d^c_H}$ can then be determined, as follows [74]

$$\ln\frac{M_{Q_H}}{m_Z} = \pi\left(\frac{1}{2\alpha_e} - \frac{1}{3\alpha_3} - \frac{\sin^2\theta_w - 0.0029}{\alpha_e}\right) - 2\ln\frac{M_U}{m_Z} - 0.63, \tag{39}$$

$$\ln\frac{M_{d^c_H}}{m_Z} = \pi\left(\frac{1}{2\alpha_e} - \frac{7}{3\alpha_3} + \frac{\sin^2\theta_w - 0.0029}{\alpha_e}\right) - 6\ln\frac{M_U}{m_Z} - 1.47, \tag{40}$$

where α_e, α_3 and $\sin^2\theta_w$ are all measured at M_Z. This is a one-loop determination (the constants account for the dominant two-loop corrections) which neglects all low-

Table 3: The value of the gap particle masses and the unified coupling for $\alpha_3(M_Z) = 0.118 \pm 0.008$. We have taken $M_U = 10^{18}\,\mathrm{GeV}$, $\sin^2\theta_w = 0.233$, and $\alpha_e^{-1} = 127.9$.

$\alpha_3(M_Z)$	$M_{d^c_H}$ (GeV)	M_{Q_H} (GeV)	$\alpha(M_U)$
0.110	4.9×10^4 GeV	2.2×10^{12} GeV	0.0565
0.118	4.5×10^6 GeV	4.1×10^{12} GeV	0.0555
0.126	2.3×10^8 GeV	7.3×10^{12} GeV	0.0547

Figure 18: The running of the gauge couplings in the $SU(5) \times U(1)$ model for $\alpha_3(M_Z) = 0.118$ (solid lines). The gap particle masses have been derived using the gauge coupling RGEs to achieve unification at $M_U = 10^{18}\,\mathrm{GeV}$. The case with no gap particles (dotted lines) is also shown; here $M_U \approx 10^{16}\,\mathrm{GeV}$.

and high-energy threshold effects,[9] but is quite adequate for our present purposes. As shown in Table 3 (and Eq. (40)) the d^c_H mass depends most sensitively on $\alpha_3(M_Z) = 0.118 \pm 0.008$ [77], whereas the Q_H mass and the unified coupling are rather insensitive to it. The unification of the gauge couplings is shown in Fig. 18 (solid lines) for the central value of $\alpha_3(M_Z)$. This figure also shows the case of no gap particles (dotted lines), for which $M_U \approx 10^{16}$ GeV.

7.2 The problem of supersymmetry breaking

A very important component of the model is that which triggers supersymmetry breaking. In string models this task is performed by the hidden sector and the universal moduli and dilaton fields. Model-dependent calculations are required to determine the precise nature of supersymmetry breaking in a given string model. In fact, no explicit string model exists to date where various theoretical difficulties (*e.g.*, suitably suppressed cosmological constant, suitable vacuum state with perturbative gauge coupling, etc.) have been shown to be satisfactorily overcome. Instead, it has become apparent [78, 79, 80] that a more model-independent approach to the problem may be more profitable. In this approach one parametrizes the breaking of supersymmetry by the largest F-term vacuum expectation value which triggers supersymmetry breaking. Of all the possible fields which could be involved (*i.e.*, hidden sector matter fields, various moduli fields, dilaton) the dilaton and three of the moduli fields are quite common in string constructions and have thus received the most attention to date. In a way, if supersymmetry breaking is triggered by these fields (*i.e.*, $\langle F_S \rangle \neq 0$ or $\langle F_T \rangle \neq 0$), this would be a rather generic prediction of string theory.

There are various possible scenarios for supersymmetry breaking which are obtained in this model-independent way. To discriminate among these we consider a simplified expression for the scalar masses (*e.g.*, $m_{\tilde{q}}$) $\widetilde{m}_i^2 = m_{3/2}^2(1 + n_i \cos^2\theta)$, with $\tan\theta = \langle F_S \rangle / \langle F_T \rangle$ [80]. Here $m_{3/2}$ is the gravitino mass and the n_i are the modular weights of the respective matter field. There are two ways in which one can obtain universal scalar masses, as strongly desired phenomenologically to avoid large flavor-changing-neutral-currents (FCNCs) [81]: (i) setting $\theta = \pi/2$, that is $\langle F_S \rangle \gg \langle F_T \rangle$; or (ii) in a model where all n_i are the same, as occurs for $Z_2 \times Z_2$ orbifolds [80] and free-fermionic constructions [82].

In the first scenario, supersymmetry breaking is triggered by the dilaton F-term and yields universal soft-supersymmetry-breaking gaugino and scalar masses and trilinear interactions [79, 80]

$$m_0 = \tfrac{1}{\sqrt{3}} m_{1/2}, \qquad A = -m_{1/2}. \tag{41}$$

This supersymmetry breaking scenario has been studied recently in the context of $SU(5) \times U(1)$ supergravity [83] and in the MSSM in Ref. [84]. In the second scenario,

[9] Here we assume a common supersymmetric threshold at M_Z. In fact, the supersymmetric threshold and the d^c_H mass are anticorrelated. See Ref. [74] for a discussion.

in the limit $\langle F_T\rangle \gg \langle F_S\rangle$ (*i.e.*, $\theta \to 0$) all scalar masses at the unification scale vanish, as is the case in no-scale supergravity models with a unified group structure [20]. In this case we have

$$m_0 = 0, \qquad A = 0. \tag{42}$$

7.3 Phenomenology: general case

The procedure to extract the low-energy predictions of the models outlined above is rather standard by now (see *e.g.*, Ref. [27]): (a) the bottom-quark and tau-lepton masses, together with the input values of m_t and $\tan\beta$ are used to determine the respective Yukawa couplings at the electroweak scale; (b) the gauge and Yukawa couplings are then run up to the unification scale $M_U = 10^{18}\,\mathrm{GeV}$ taking into account the extra vector-like quark doublet ($\sim 10^{12}\,\mathrm{GeV}$) and singlet ($\sim 10^{6}\,\mathrm{GeV}$) introduced above [74, 68]; (c) at the unification scale the soft-supersymmetry breaking parameters are introduced (according to Eqs. (42,41)) and the scalar masses are then run down to the electroweak scale; (d) radiative electroweak symmetry breaking is enforced by minimizing the one-loop effective potential which depends on the whole mass spectrum, and the values of the Higgs mixing term $|\mu|$ and the bilinear soft-supersymmetry breaking parameter B are determined from the minimization conditions; (e) all known phenomenological constraints on the sparticle and Higgs masses are applied (most importantly the LEP lower bounds on the chargino and Higgs masses), including the cosmological requirement of not-too-large neutralino relic density.

7.3.1 Mass ranges

We have scanned the parameter space for $m_t = 130, 150, 170\,\mathrm{GeV}$, $\tan\beta = 2 \to 50$ and $m_{1/2} = 50 \to 500\,\mathrm{GeV}$. Imposing the constraint $m_{\tilde{g},\tilde{q}} < 1\,\mathrm{TeV}$ we find

$$\langle F_M\rangle_{m_0=0}: \qquad m_{1/2} < 475\,\mathrm{GeV}, \quad \tan\beta \lesssim 32, \tag{43}$$

$$\langle F_D\rangle: \qquad m_{1/2} < 465\,\mathrm{GeV}, \quad \tan\beta \lesssim 46. \tag{44}$$

These restrictions on $m_{1/2}$ cut off the growth of most of the sparticle and Higgs masses at $\approx 1\,\mathrm{TeV}$. However, the sleptons, the lightest Higgs, the two lightest neutralinos, and the lightest chargino are cut off at a much lower mass, as follows[10]

$$\langle F_M\rangle_{m_0=0}: \quad \begin{cases} m_{\tilde{e}_R} < 190\,\mathrm{GeV}, \quad m_{\tilde{e}_L} < 305\,\mathrm{GeV}, \quad m_{\tilde{\nu}} < 295\,\mathrm{GeV} \\ m_{\tilde{\tau}_1} < 185\,\mathrm{GeV}, \quad m_{\tilde{\tau}_2} < 315\,\mathrm{GeV} \\ m_h < 125\,\mathrm{GeV} \\ m_{\chi^0_1} < 145\,\mathrm{GeV}, \quad m_{\chi^0_2} < 290\,\mathrm{GeV}, \quad m_{\chi^\pm_1} < 290\,\mathrm{GeV} \end{cases} \tag{45}$$

[10] In this class of supergravity models the three sneutrinos ($\tilde{\nu}$) are degenerate in mass. Also, $m_{\tilde{\mu}_L} = m_{\tilde{e}_L}$ and $m_{\tilde{\mu}_R} = m_{\tilde{e}_R}$.

Table 4: The value of the c_i coefficients appearing in Eq. (28), the ratio $c_{\tilde{g}} = m_{\tilde{g}}/m_{1/2}$, and the average squark coefficient $\bar{c}_{\tilde{q}}$, for $\alpha_3(M_Z) = 0.118 \pm 0.008$. Also shown are the a_i, b_i coefficients for the central value of $\alpha_3(M_Z)$ and both supersymmetry breaking scenaria ($M : \langle F_M \rangle_{m_0=0}$, $D : \langle F_D \rangle$). The results apply as well to the second-generation squark and slepton masses.

i	$c_i\,(0.110)$	$c_i\,(0.118)$	$c_i\,(0.126)$
$\tilde{\nu}, \tilde{e}_L$	0.406	0.409	0.413
$\tilde{e}_R$	0.153	0.153	0.153
$\tilde{u}_L, \tilde{d}_L$	3.98	4.41	4.97
$\tilde{u}_R$	3.68	4.11	4.66
$\tilde{d}_R$	3.63	4.06	4.61
$c_{\tilde{g}}$	1.95	2.12	2.30
$\bar{c}_{\tilde{q}}$	3.82	4.07	4.80

i	$a_i(M)$	$b_i(M)$	$a_i(D)$	$b_i(D)$
$\tilde{e}_L$	0.302	+1.115	0.406	+0.616
$\tilde{e}_R$	0.185	+2.602	0.329	+0.818
$\tilde{\nu}$	0.302	−2.089	0.406	−1.153
$\tilde{u}_L$	0.991	−0.118	1.027	−0.110
$\tilde{u}_R$	0.956	−0.016	0.994	−0.015
$\tilde{d}_L$	0.991	+0.164	1.027	+0.152
$\tilde{d}_R$	0.950	−0.033	0.989	−0.030

$$\langle F_D \rangle : \qquad \left\{ \begin{array}{l} m_{\tilde{e}_R} < 325\,\mathrm{GeV}, \quad m_{\tilde{e}_L} < 400\,\mathrm{GeV}, \quad m_{\tilde{\nu}} < 400\,\mathrm{GeV} \\ m_{\tilde{\tau}_1} < 325\,\mathrm{GeV}, \quad m_{\tilde{\tau}_2} < 400\,\mathrm{GeV} \\ m_h < 125\,\mathrm{GeV} \\ m_{\chi^0_1} < 145\,\mathrm{GeV}, \quad m_{\chi^0_2} < 285\,\mathrm{GeV}, \quad m_{\chi^\pm_1} < 285\,\mathrm{GeV} \end{array} \right. \tag{46}$$

It is interesting to note that due to the various constraints on the model, the gluino and (average) squark masses are bounded from below,

$$\langle F_M \rangle_{m_0=0} : \left\{ \begin{array}{l} m_{\tilde{g}} \gtrsim 245\,(260)\,\mathrm{GeV} \\ m_{\tilde{q}} \gtrsim 240\,(250)\,\mathrm{GeV} \end{array} \right. \qquad \langle F_D \rangle : \left\{ \begin{array}{l} m_{\tilde{g}} \gtrsim 195\,(235)\,\mathrm{GeV} \\ m_{\tilde{q}} \gtrsim 195\,(235)\,\mathrm{GeV} \end{array} \right. \tag{47}$$

for $\mu > 0 (\mu < 0)$. Relaxing the above conditions on $m_{1/2}$ simply allows all sparticle masses to grow further proportional to $m_{\tilde{g}}$.

7.3.2 Mass relations

The neutralino and chargino masses show a correlation observed before in this class of models [65, 68], namely

$$m_{\chi^0_1} \approx \tfrac{1}{2} m_{\chi^0_2}, \qquad m_{\chi^0_2} \approx m_{\chi^\pm_1} \approx M_2 = (\alpha_2/\alpha_3) m_{\tilde{g}} \approx 0.28 m_{\tilde{g}}. \tag{48}$$

This is because throughout the parameter space $|\mu|$ is generally much larger than M_W and $|\mu| > M_2$. In practice we find $m_{\chi^0_2} \approx m_{\chi^\pm_1}$ to be satisfied quite accurately, whereas $m_{\chi^0_1} \approx \frac{1}{2} m_{\chi^0_2}$ is only qualitatively satisfied, although the agreement is better in the $\langle F_D \rangle$ case. In fact, these two mass relations are much more reliable than the one that links them to $m_{\tilde{g}}$. The heavier neutralino ($\chi^0_{3,4}$) and chargino ($\chi^\pm_2$) masses are determined by the value of $|\mu|$; they all approach this limit for large enough $|\mu|$.

More precisely, $m_{\chi_3^0}$ approaches $|\mu|$ sooner than $m_{\chi_4^0}$ does. On the other hand, $m_{\chi_4^0}$ approaches $m_{\chi_2^\pm}$ rather quickly.

The first- and second-generation squark and slepton masses can be determined analytically

$$\widetilde{m}_i = \left[m_{1/2}^2(c_i+\xi_0^2) - d_i\frac{\tan^2\beta-1}{\tan^2\beta+1}M_W^2\right]^{1/2} = a_i m_{\tilde g}\left[1+b_i\left(\frac{150}{m_{\tilde g}}\right)^2\frac{\tan^2\beta-1}{\tan^2\beta+1}\right]^{1/2}, \tag{49}$$

where $d_i=(T_{3i}-Q)\tan^2\theta_w+T_{3i}$ (*e.g.*, $d_{\tilde u_L}=\frac{1}{2}-\frac{1}{6}\tan^2\theta_w$, $d_{\tilde e_R}=-\tan^2\theta_w$), and $\xi_0=m_0/m_{1/2}=0,\frac{1}{\sqrt{3}}$. The coefficients c_i can be calculated numerically in terms of the low-energy gauge couplings, and are given in Table 4[11] for $\alpha_3(M_Z)=0.118\pm0.008$. In the table we also give $c_{\tilde g}=m_{\tilde g}/m_{1/2}$. Note that these values are smaller than what is obtained in the $SU(5)$ supergravity model (where $c_{\tilde g}=2.90$ for $\alpha_3(M_Z)=0.118$) and therefore the numerical relations between the gluino mass and the neutralino masses are different in that model. In the table we also show the resulting values for a_i, b_i for the central value of $\alpha_3(M_Z)$. Note that the apparently larger $\tan\beta$ dependence in the $\langle F_M\rangle_{m_0=0}$ case (*i.e.*, $|b_i(M)|>|b_i(D)|$) is actually compensated by a larger minimum value of $m_{\tilde g}$ in this case (see Eq. (47)).

The "average" squark mass, $m_{\tilde q}\equiv\frac{1}{8}(m_{\tilde u_L}+m_{\tilde u_R}+m_{\tilde d_L}+m_{\tilde d_R}+m_{\tilde c_L}+m_{\tilde c_R}+m_{\tilde s_L}+m_{\tilde s_R})=(m_{\tilde g}/c_{\tilde q})\sqrt{\bar c_{\tilde q}+\xi_0^2}$, with $\bar c_{\tilde q}$ given in Table 4, is determined to be

$$m_{\tilde q}=\begin{cases}(1.00,0.95,0.95)m_{\tilde g}, & \langle F_M\rangle_{m_0=0}\\ (1.05,0.99,0.98)m_{\tilde g}, & \langle F_D\rangle\end{cases} \tag{50}$$

for $\alpha_3(M_Z)=0.110,0.118,0.126$ (the dependence on $\tan\beta$ is small). The squark splitting around the average is $\approx 2\%$.

These masses are plotted in Fig. 19. The thickness and straightness of the lines shows the small $\tan\beta$ dependence, except for $\tilde\nu$. The results do not depend on the sign of μ, except to the extent that some points in parameter space are not allowed for both signs of μ: the $\mu<0$ lines start-off at larger mass values. Note that

$$\langle F_M\rangle_{m_0=0}:\begin{cases}m_{\tilde e_R}\approx 0.18m_{\tilde g}\\ m_{\tilde e_L}\approx 0.30m_{\tilde g}\\ m_{\tilde e_R}/m_{\tilde e_L}\approx 0.61\end{cases}\qquad \langle F_D\rangle:\begin{cases}m_{\tilde e_R}\approx 0.33m_{\tilde g}\\ m_{\tilde e_L}\approx 0.41m_{\tilde g}\\ m_{\tilde e_R}/m_{\tilde e_L}\approx 0.81\end{cases} \tag{51}$$

The third generation squark and slepton masses cannot be determined analytically. In Fig. 20 we show $\tilde\tau_{1,2},\tilde b_{1,2},\tilde t_{1,2}$ for the choice $m_t=150\,\mathrm{GeV}$. The variability on the $\tilde\tau_{1,2}$ and $\tilde b_{1,2}$ masses is due to the $\tan\beta$-dependence in the off-diagonal element of the corresponding 2×2 mass matrices ($\propto m_{\tau,b}(A_{\tau,b}+\mu\tan\beta)$). The off-diagonal element in the stop-squark mass matrix ($\propto m_t(A_t+\mu/\tan\beta)$) is rather insensitive to $\tan\beta$ but still effects a large $\tilde t_1-\tilde t_2$ mass splitting because of the significant A_t

[11] These are renormalized at the scale M_Z. In a more accurate treatment, the c_i would be renormalized at the physical sparticle mass scale, leading to second order shifts on the sparticle masses.

Figure 19: The first-generation squark and slepton masses as a function of the gluino mass, for both signs of μ, $m_t = 150\,\mathrm{GeV}$, and both supersymmetry breaking scenaria under consideration. The same values apply to the second generation. The thickness of the lines and their deviation from linearity are because of the small $\tan\beta$ dependence.

contribution. Note that both these effects are more pronounced for the $\langle F_D \rangle$ case since there $|A_{t,b,\tau}|$ are larger than in the $\langle F_M \rangle_{m_0=0}$ case. The lowest values of the $\tilde{t}_1$ mass go up with m_t and can be as low as

$$m_{\tilde{t}_1} \gtrsim \begin{cases} 160, 170, 190\,(155, 150, 170)\,\mathrm{GeV}; & \langle F_M \rangle_{m_0=0} \\ 88, 112, 150\,(92, 106, 150)\,\mathrm{GeV}; & \langle F_D \rangle \end{cases} \tag{52}$$

for $m_t = 130, 150, 170\,\mathrm{GeV}$ and $\mu > 0\,(\mu < 0)$.

The one-loop corrected lightest CP-even (h) and CP-odd (A) Higgs boson masses are shown in Fig. 21 as functions of $m_{\tilde{g}}$ for $m_t = 150\,\mathrm{GeV}$. Following the methods of Ref. [66] we have determined that the LEP lower bound on m_h becomes $m_h \gtrsim 60\,\mathrm{GeV}$, as the figure shows. The largest value of m_h depends on m_t; we find

$$m_h < \begin{cases} 106, 115, 125\,\mathrm{GeV}; & \langle F_M \rangle_{m_0=0} \\ 107, 117, 125\,\mathrm{GeV}; & \langle F_D \rangle \end{cases} \tag{53}$$

for $m_t = 130, 150, 170\,\mathrm{GeV}$. It is interesting to note that the one-loop corrected values of m_h for $\tan\beta = 2$ are quite dependent on the sign of μ. This phenomenon can be traced back to the $\tilde{t}_1 - \tilde{t}_2$ mass splitting which enhances the dominant $\tilde{t}$ one-loop corrections to m_h [85], an effect which is usually neglected in phenomenological analyses. The $\tilde{t}_{1,2}$ masses for $\tan\beta = 2$ are drawn closer together than the rest. The opposite effect occurs for $\mu < 0$ and therefore the one-loop correction is larger in this case. The sign-of-μ dependence appears in the off-diagonal entries in the $\tilde{t}$ mass matrix $\propto m_t(A_t + \mu/\tan\beta)$, with $A_t < 0$ in this case. Clearly only small $\tan\beta$ matters, and $\mu < 0$ enhances the splitting. The A-mass grows fairly linearly with $m_{\tilde{g}}$ with a $\tan\beta$-dependent slope which decreases for increasing $\tan\beta$, as shown in Fig. 21. Note that even though m_A can be fairly light, we always get $m_A > m_h$, in agreement with a general theorem to this effect in supergravity theories [86]. This result also implies that the channel $e^+e^- \to hA$ at LEPI is not kinematically allowed in this model.

7.3.3 Neutralino relic density

The computation of the neutralino relic density (following the methods of Refs. [57, 58]) shows that $\Omega_\chi h_0^2 \lesssim 0.25\,(0.90)$ in the no-scale (dilaton) model. This implies that in these models the cosmologically interesting values $\Omega_\chi h_0^2 \lesssim 1$ occur quite naturally. These results are in good agreement with the observational upper bound on $\Omega_\chi h_0^2$ [60]. Moreover, fits to the COBE data and the small and large scale structure of the Universe suggest [87] a mixture of $\approx 70\%$ cold dark matter and $\approx 30\%$ hot dark matter together with $h_0 \approx 0.5$. The hot dark matter component in the form of massive tau neutrinos has already been shown to be compatible with the $SU(5) \times U(1)$ model we consider here [73, 75], whereas the cold dark matter component implies $\Omega_\chi h_0^2 \approx 0.17$ which is reachable in these models.

Figure 20: The $\tilde{\tau}_{1,2}$, $\tilde{b}_{1,2}$, and $\tilde{t}_{1,2}$ masses versus the gluino mass for both signs of μ, $m_t = 150\,\mathrm{GeV}$, and both supersymmetry breaking scenaria. The variability in the $\tilde{\tau}_{1,2}$, $\tilde{b}_{1,2}$, and $\tilde{t}_{1,2}$ masses is because of the off-diagonal elements of the corresponding mass matrices.

Figure 21: The one-loop corrected h and A Higgs masses versus the gluino mass for both signs of μ, $m_t = 150\,\mathrm{GeV}$, and the two supersymmetry breaking scenaria. Representative values of $\tan\beta$ are indicated.

7.4 Phenomenology: special cases

7.4.1 The strict no-scale case: a striking result

We now impose the additional constraint $B(M_U) = 0$ to be added to Eq. (42), and obtain the so-called strict no-scale case [68]. Since $B(M_Z)$ is determined by the radiative electroweak symmetry breaking conditions, this added constraint needs to be imposed in a rather indirect way. That is, for given $m_{\tilde{g}}$ and m_t values, we scan the possible values of $\tan\beta$ looking for cases where $B(M_U) = 0$. The most striking result is that solutions exist *only* for $m_t \lesssim 135\,\mathrm{GeV}$ if $\mu > 0$ and for $m_t \gtrsim 140\,\mathrm{GeV}$ if $\mu < 0$. That is, the value of m_t *determines* the sign of μ. Furthermore, for $\mu < 0$ the value of $\tan\beta$ is determined uniquely as a function of m_t and $m_{\tilde{g}}$, whereas for $\mu > 0$, $\tan\beta$ can be double-valued for some m_t range which includes $m_t = 130\,\mathrm{GeV}$ (but does not include $m_t = 100\,\mathrm{GeV}$). In Fig. 22 (top row) we plot the solutions found in this manner for the indicated m_t values.

All the mass relationships deduced in the previous section apply here as well. The $\tan\beta$-spread that some of them have will be much reduced though. The most noticeable changes occur for the quantities which depend most sensitively on $\tan\beta$. In Fig. 22 (bottom row) we plot the one-loop corrected lightest Higgs boson mass versus $m_{\tilde{g}}$. The result is that m_h is basically determined by m_t; only a weak dependence on $m_{\tilde{g}}$ exists. Moreover, for $m_t \lesssim 135\,\mathrm{GeV} \Leftrightarrow \mu > 0$, $m_h \lesssim 105\,\mathrm{GeV}$; whereas for $m_t \gtrsim 140\,\mathrm{GeV} \Leftrightarrow \mu < 0$, $m_h \gtrsim 100\,\mathrm{GeV}$. Therefore, in the strict no-scale case, once the top-quark mass is measured, we will know the sign of μ and whether m_h is above or below $100\,\mathrm{GeV}$.

For $\mu > 0$, we just showed that the strict no-scale constraint requires $m_t \lesssim 135\,\mathrm{GeV}$. This implies that μ cannot grow as large as it did previously in the general case. In fact, for $\mu > 0$, $\mu_{max} \approx 745\,\mathrm{GeV}$ before and $\mu_{max} \approx 440\,\mathrm{GeV}$ now. This smaller value of μ_{max} has the effect of cutting off the growth of the $\chi^0_{3,4}, \chi^\pm_2$ masses at $\approx \mu_{max} \approx 440\,\mathrm{GeV}$ (c.f. $\approx 750\,\mathrm{GeV}$) and of the heavy Higgs masses at $\approx 530\,\mathrm{GeV}$ (c.f. $\approx 940\,\mathrm{GeV}$).

7.4.2 The special dilaton scenario case

In our analysis above, the radiative electroweak breaking conditions were used to determine the magnitude of the Higgs mixing term μ at the electroweak scale. This quantity is ensured to remain light as long as the supersymmetry breaking parameters remain light. In a fundamental theory this parameter should be calculable and its value used to determine the Z-boson mass. From this point of view it is not clear that the natural value of μ should be light. In specific models one can obtain such values by invoking non-renormalizable interactions [88, 71, 89]. Another contribution to this quantity is generically present in string supergravity models [90, 89, 79]. The general case with contributions from both sources has been effectively dealt with in the previous section. If one assumes that only supergravity-induced contributions to μ exist, then it can be shown that the B-parameter at the unification scale is also

Figure 22: The value of $\tan\beta$ versus $m_{\tilde{g}}$ in the strict no-scale case (where $B(M_U)=0$) for the indicated values of m_t. Note that the sign of μ is *determined* by m_t and that $\tan\beta$ can be double-valued for $\mu>0$. Also shown is the one-loop corrected lightest Higgs boson mass. Note that if $\mu>0$ (for $m_t<135\,\mathrm{GeV}$) then $m_h<105\,\mathrm{GeV}$; whereas if $\mu<0$ (for $m_t>140\,\mathrm{GeV}$) then $m_h>100\,\mathrm{GeV}$.

determined [79, 80],

$$B(M_U)=2m_0=\tfrac{2}{\sqrt{3}}m_{1/2}, \tag{54}$$

which is to be added to the set of relations in Eq. (41). This new constraint effectively determines $\tan\beta$ for given m_t and $m_{\tilde{g}}$ values and makes this restricted version of the model highly predictive [83].

From the outset we note that only solutions with $\mu<0$ exist. This is not a completely obvious result, but it can be partially understood as follows. In tree-level approximation, $m_A^2>0\Rightarrow\mu B<0$ at the electroweak scale. Since $B(M_U)$ is required to be positive and not small, $B(M_Z)$ will likely be positive also, thus forcing μ to be negative. A sufficiently small value of $B(M_U)$ and/or one-loop corrections to m_A^2 could alter this result, although in practice this does not happen. A numerical iterative procedure allows us to determine the value of $\tan\beta$ which satisfies Eq. (54), from the calculated value of $B(M_Z)$. We find that

$$\tan\beta\approx 1.57-1.63, 1.37-1.45, 1.38-1.40 \quad \text{for } m_t=130,150,155\,\mathrm{GeV} \tag{55}$$

is required. Since $\tan\beta$ is so small ($m_h^{tree}\approx 28-41\,\mathrm{GeV}$), a significant one-loop

Table 5: The range of allowed sparticle and Higgs masses in the restricted dilaton scenario. The top-quark mass is restricted to be $m_t < 155\,\mathrm{GeV}$. All masses in GeV.

m_t	130	150	155
$\tilde{g}$	$335-1000$	$260-1000$	$640-1000$
χ_1^0	$38-140$	$24-140$	$90-140$
$\chi_2^0, \chi_1^\pm$	$75-270$	$50-270$	$170-270$
$\tan\beta$	$1.57-1.63$	$1.37-1.45$	$1.38-1.40$
h	$61-74$	$64-87$	$84-91$
$\tilde{l}$	$110-400$	$90-400$	$210-400$
$\tilde{q}$	$335-1000$	$260-1000$	$640-1000$
A, H, H^+	>400	>400	>970

correction to m_h is required to increase it above its experimental lower bound of $\approx 60\,\mathrm{GeV}$ [66]. This requires the largest possible top-quark masses (and a not-too-small squark mass). However, perturbative unification imposes an upper bound on m_t for a given $\tan\beta$ [91], which in this case implies [27]

$$m_t \lesssim 155\,\mathrm{GeV}, \tag{56}$$

which limits the magnitude of m_h

$$m_h \lesssim 74, 87, 91\,\mathrm{GeV} \qquad \text{for} \qquad m_t = 130, 150, 155\,\mathrm{GeV}. \tag{57}$$

Lower values of m_t are experimentally disfavored.

In Table 5 we give the range of sparticle and Higgs masses that are allowed in this case. Clearly, continuing top-quark searches at the Tevatron and Higgs searches at LEPI,II should probe this restricted scenario completely.

8 Detailed calculations for the Tevatron

The sparticle and Higgs spectrum discussed so far can be directly explored partially at present and near future collider facilities, as we now discuss for each model considered above. Let us start with the Tevatron, where the main topics of experimental interest are:

(a) The search and eventual discovery of the top quark will narrow down the parameter space of these models considerably. Moreover, in the two special $SU(5) \times U(1)$ cases discussed in Sec. 7.4 this measurement will be very important: (i) in the strict no-scale case (Sec. 7.4.1) it will determine the sign of μ ($\mu > 0$ if $m_t \lesssim 135\,\mathrm{GeV}$; $\mu < 0$ if $m_t \gtrsim 140\,\mathrm{GeV}$) and whether the Higgs mass is above or below $\approx 100\,\mathrm{GeV}$, and (ii) it may rule out the restricted dilaton scenario (Sec. 7.4.2) if $m_t > 150\,\mathrm{GeV}$.

(b) The trilepton signal in $p\bar{p} \to \chi_2^0 \chi_1^\pm X$, where χ_2^0 and $\chi_1^\pm$ both decay leptonically, is a clean test of supersymmetry [92] and in particular of this class of models [67]. Examples of trilepton rates in the no-scale $SU(5) \times U(1)$ and in the $SU(5)$ model are shown in Fig. 23. For more details see Ref. [67]. With $\mathcal{L} = 100\,\mathrm{pb}^{-1}$ of integrated luminosity basically all of the parameter space of the $SU(5)$ model should be explorable. Also, chargino masses as high as $\approx 175\,\mathrm{GeV}$ in the no-scale model could be explored, although some regions of parameter space for lighter chargino masses would remain unexplored. We expect that somewhat weaker results will hold for the dilaton model, since the sparticle masses are heavier in that model, especially the sleptons which enhance the leptonic branching ratios when they are light enough [67].

(c) The relation $m_{\tilde{q}} \approx m_{\tilde{g}}$ for the $\tilde{u}_{L,R}, \tilde{d}_{L,R}$ squark masses in the $SU(5)\times U(1)$ models should allow to probe the low end of the squark and gluino allowed mass ranges, although the outlook is more promising for the dilaton model since the allowed range starts off at lower values of $m_{\tilde{g},\tilde{q}}$ (see Eq. (47)). An important point distinguishing the two models is that the average squark mass is slightly below (above) the gluino mass in the no-scale (dilaton) model, which should have an important bearing on the experimental signatures and rates [93]. In the dilaton case the $\tilde{t}_1$ mass can be below $100\,\mathrm{GeV}$ for sufficiently low m_t, and thus may be detectable. As the lower bound on m_t rises, this signal becomes less accessible. The actual reach of the Tevatron for the above processes depends on its ultimate integrated luminosity. The squark masses in the $SU(5)$ model ($m_{\tilde{q}} \gtrsim 500\,\mathrm{GeV}$) are beyond the reach of the Tevatron.

Very recently the first limits from the Tevatron on trilepton searches have been announced, as discussed in (b) above. These limits [94], along with the experimental predictions are shown in Fig. 24. At present no useful constraints on the parameter space can be obtained from these searches. However, with the expected increase in luminosity by a factor of four during 1994, it should be possible to probe regions of parameter space with chargino masses as large as 100 GeV.

9 Detailed calculations for LEP

9.1 LEP I

The current LEPI lower bound on the Standard Model (SM) Higgs boson mass ($m_H > 62\,\mathrm{GeV}$ [95]) is obtained by studying the process $e^+e^- \to Z^*H$ with subsequent Higgs decay into two jets. The MSSM analog of this production process leads to a cross section differing just by a factor of $\sin^2(\alpha - \beta)$, where α is the SUSY Higgs mixing angle and $\tan\beta = v_2/v_1$ is the ratio of the Higgs vacuum expectation values [96]. The published LEPI lower bound on the lightest SUSY Higgs boson mass ($m_h > 43\,\mathrm{GeV}$) is the result of allowing $\sin^2(\alpha - \beta)$ to vary throughout the MSSM parameter space and by considering the $e^+e^- \to Z^*h, hA$ cross sections. It is therefore possible that in

Figure 23: The number of trilepton events at the Tevatron per $100\,\mathrm{pb}^{-1}$ in the $SU(5)$ model ("minimal $SU(5)$ in the figure) and in no-scale $SU(5)\times U(1)$ (for $m_t = 130\,\mathrm{GeV}$). Note that with $200\,\mathrm{pb}^{-1}$ and 60% detection efficiency it should be possible to probe basically all of the parameter space of the $SU(5)$ model, and probe chargino masses as high as $175\,\mathrm{GeV}$ in the no-scale model.

Figure 24: The trilepton cross section at the Tevatron ($p\bar{p} \to \chi_2^0 \chi_1^\pm X$; $\chi_2^0 \to \chi_1^0 l^+ l^-$, $\chi_1^\pm \to \chi_1^0 l^\pm \nu_l$, with $l = e, \mu$) in $SU(5) \times U(1)$ supergravity for both no-scale and dilaton scenarios. The CDF 95%CL upper limit is shown (solid line), as well a possible improved limit at the end of 1994 (dashed line).

specific models (which embed the MSSM), where $\sin^2(\alpha - \beta)$ is naturally restricted to be near unity, the lower bound on m_h could rise, and even reach the SM lower bound if $\mathrm{BR}(h \to 2\,\mathrm{jets})$ is SM-like as well. This has been shown to be the case for the supergravity models we discuss here, and more generally for supergravity models which enforce radiative electroweak symmetry breaking [66].

Non-observation of a SM Higgs signal puts the following upper bound in the number of expected 2-jet events.

$$\#\mathrm{events}_{\mathrm{SM}} = \sigma(e^+e^- \to Z^* H)_{\mathrm{SM}} \times \mathrm{BR}(H \to 2\,\mathrm{jets})_{\mathrm{SM}} \times \int \mathcal{L} dt < 3. \tag{58}$$

The SM value for $\mathrm{BR}(H \to 2\,\mathrm{jets})_{\mathrm{SM}} \approx \mathrm{BR}(H \to b\bar{b} + c\bar{c} + gg)_{\mathrm{SM}} \approx 0.92$ [96] corresponds to an upper bound on $\sigma(e^+e^- \to Z^* H)_{\mathrm{SM}}$. Since this is a monotonically decreasing function of m_H, a lower bound on m_H follows, *i.e.*, $m_H > 62\,\mathrm{GeV}$ as noted above. We denote by $\sigma_{\mathrm{SM}}(62)$ the corresponding value for $\sigma(e^+e^- \to Z^* H)_{\mathrm{SM}}$. For the MSSM the following relations hold

$$\sigma(e^+e^- \to Z^* h)_{\mathrm{SUSY}} = \sin^2(\alpha - \beta)\sigma(e^+e^- \to Z^* H)_{\mathrm{SM}}, \tag{59}$$

$$\mathrm{BR}(h \to 2\,\mathrm{jets})_{\mathrm{SUSY}} = f \cdot \mathrm{BR}(H \to 2\,\mathrm{jets})_{\mathrm{SM}}. \tag{60}$$

From Eq. (58) one can deduce the integrated luminosity achieved, $\int \mathcal{L}dt = 3/(\sigma_{\rm SM}(62){\rm BR}_{\rm SM})$. In analogy with Eq. (58), we can write

$$\#{\rm events}_{\rm SUSY} = \sigma_{\rm SUSY}(m_h) \times {\rm BR}_{\rm SUSY} \times \int \mathcal{L}dt = 3f \cdot \sigma_{\rm SUSY}(m_h)/\sigma_{\rm SM}(62) < 3. \quad (61)$$

This immediately implies the following condition for *allowed* points in parameter space [66, 97]

$$f \cdot \sin^2(\alpha - \beta) < P(62/M_Z)/P(m_h/M_Z), \quad (62)$$

where we have used the fact that the cross sections differ simply by the coupling factor $\sin^2(\alpha-\beta)$ and the Higgs mass dependence which enters through a function P [96]

$$\begin{aligned} P(y) &= \frac{3y(y^4-8y^2+20)}{\sqrt{4-y^2}} \cos^{-1}\left(\frac{y(3-y^2)}{2}\right) - 3(y^4-6y^2+4)\ln y \\ & -\tfrac{1}{2}(1-y^2)(2y^4-13y+47)\,. \end{aligned} \quad (63)$$

The cross section $\sigma_{\rm SUSY}(m_h)$ for the $SU(5)$ model also corresponds to the SM result since one can verify that $\sin^2(\alpha-\beta) > 0.9999$ in this case. For the flipped model there is a small deviation ($\sin^2(\alpha-\beta) > 0.95$) relative to the SM result for some points [66]. In the calculation of ${\rm BR}(h \to 2\,{\rm jets})_{\rm SUSY}$ which enters in the ratio f, we have included *all* contributing modes, in particular the invisible $h \to \chi_1^0\chi_1^0$ decays. The conclusion is that these models differ little from the SM and in fact the proper lower bound on m_h is very near 60 GeV, although it varies from point to point in the parameter space.

In Ref. [66] it was also shown that this phenomenon is due to a decoupling effect of the Higgs sector as the supersymmetry scale rises, and it is communicated to the Higgs sector through the radiative electroweak symmetry breaking mechanism. The point to be stressed is that if the supersymmetric Higgs sector is found to be SM-like, this could be taken as *indirect* evidence for an underlying radiative electroweak breaking mechanism, since no insight could be garnered from the MSSM itself.

Note that since the lower bound on the SM Higgs boson mass could still be pushed up several GeV at LEPI, the strict dilaton scenario in Sec. 7.4.2 (which requires $m_h \approx 61-91\,{\rm GeV}$) could be further constrained at LEPI.

9.2 LEP II

(a) At LEPII the SM Higgs mass could be explored up to roughly the beam energy minus 100 GeV [98]. This will allow exploration of almost all of the Higgs parameter space in the $SU(5)$ model [99]. For $SU(5)\times U(1)$ supergravity, only low $\tan\beta$ values could be explored, although the strict no-scale case will probably be out of reach (see Figs. 21,22). The $e^+e^- \to hA$ channel will be open for $SU(5)\times U(1)$ for large $\tan\beta$ and low $m_{\tilde{g}}$. This channel is always closed in the $SU(5)$ case (since $m_A \gtrsim 1\,{\rm TeV}$). It is important to point out that the

Figure 25: The number of "mixed" events (1-lepton+2jets+$\not{p}$) events per $\mathcal{L} = 100\,\mathrm{pb}^{-1}$ at LEPII versus the chargino mass in the $SU(5)$ model.

Figure 26: The number of "mixed" events (1-lepton+2jets+$\not{p}$) events per $\mathcal{L} = 100\,\mathrm{pb}^{-1}$ at LEPII versus the chargino mass in the no-scale model (top row). Also shown (bottom row) are the number of di-electron events per $\mathcal{L} = 100\,\mathrm{pb}^{-1}$ from selectron pair production versus the lightest selectron mass.

preferred $h \to b\bar{b}, c\bar{c}, gg$ detection modes may be suppressed because of invisible Higgs decays ($h \to \chi_1^0\chi_1^0$) for $m_h \lesssim 80\,\mathrm{GeV}$ ($m_h \gtrsim 80\,\mathrm{GeV}$) by as much as 30%/15% (40%/40%) in the $SU(5)/SU(5) \times U(1)$ model [99].

(b) Chargino masses below the kinematical limit ($m_{\chi_1^\pm} \lesssim 100\,\mathrm{GeV}$) should not be a problem to detect through the "mixed" mode with one chargino decaying leptonically and the other one hadronically [99], *i.e.*, $e^+e^- \to \chi_1^+\chi_1^-$, $\chi_1^+ \to \chi_1^0 q\bar{q}'$, $\chi_1^- \to \chi_1^0 l^-\bar{\nu}_l$. In Fig. 25 and Fig. 26 (top row) we show the correponding event rates in the $SU(5)$ and no-scale $SU(5)\times U(1)$ models. Recall that $m_{\chi_1^\pm}$ can be as high as $\approx 290\,\mathrm{GeV}$ in $SU(5)\times U(1)$ supergravity, whereas $m_{\chi_1^\pm} \lesssim 100\,\mathrm{GeV}$ in the $SU(5)$ model. Interestingly enough, the number of mixed events do not overlap (they are much higher in the $SU(5)$ model) and therefore, if $m_{\chi_1^\pm} < 100\,\mathrm{GeV}$ then LEPII should be able to exclude at least one of the models.

(c) Selectron, smuon, and stau pair production is partially accessible for both cases of $SU(5)\times U(1)$ supergravity, no-scale and dilaton (although more so in the no-scale case), and completely inaccessible in the $SU(5)$ case. In Fig. 26 (bottom row) we show the rates for the most promising (dielectron) mode in $e^+e^- \to \tilde{e}_R^+\tilde{e}_R^-$ production in the no-scale model.

10 Detailed calculations for HERA

The elastic and deep-inelastic contributions to $e^-p \to \tilde{e}_R^-\chi_1^0$ and $e^-p \to \tilde{\nu}\chi_1^-$ at HERA in the no-scale $SU(5)\times U(1)$ model should push the LEPI lower bounds on the lightest selectron, the lightest neutralino, and the sneutrino masses by $\approx 25\,\mathrm{GeV}$ with $\mathcal{L} = 100\,\mathrm{pb}^{-1}$ [100]. In Fig. 27 we show the elastic plus deep-inelastic contributions to the total supersymmetric signal ($ep \to \mathrm{susy} \to eX + \not{p}$) versus the lightest selectron mass ($m_{\tilde{e}_R}$) and the sneutrino mass ($m_{\tilde{\nu}}$) in the no-scale model. These figures show the "reach" of HERA in each of these variables. With $\mathcal{L} = 1000\,\mathrm{pb}^{-1}$ HERA should be competitive with LEPII as far as the no-scale model is concerned. In the dilaton scenario, because of the somewhat heavier sparticle masses, the effectiveness of HERA is reduced, although probably both channels may be accessible. HERA is not sensitive to the $SU(5)$ model spectrum.

For elastic processes, another measurable signal is the slowed down outgoing proton. Since the transverse momentum of the outgoing proton is very small, the relative energy loss of the proton energy $z = (E_p^{in} - E_p^{out})/E_p^{in}$ is given by $z = 1 - x_L$, where x_L is the longitudinal momentum of the leading proton. It has been pointed out [101] that the z-distribution is peaked at a value not much larger than its minimal value,

$$z_{min} = \frac{1}{s}(m_{\tilde{e}_{R,L}} + m_{\chi_{1,2}^0})^2. \tag{64}$$

Therefore, the smallest measured value in the z-distribution should be a good approximation to z_{min}. Since the Leading Proton Spectrometer (LPS) of the ZEUS detector

Figure 27: The elastic plus deep-inelastic total supersymmetric cross section at HERA ($ep \to$ susy $\to eX + \not{p}$) versus the lightest selectron mass ($m_{\tilde{e}_R}$) and the sneutrino mass ($m_{\tilde{\nu}}$). The short- and long-term limits of sensitivity are expected to be 10^{-2} pb and 10^{-3} pb respectively.

at HERA can measure this distribution accurately, one may have a new way of probing the supersymmetric spectrum, as follows. We calculate the average $\tilde{z}_{min}$ weighed by the different elastic cross sections $\sigma(\tilde{e}_{R,L}\chi^0_{1,2})$. The results are shown in the top row of Fig. 28 versus the total elastic cross section. These plots show the possible values of $\tilde{z}_{min}$ for a given sensitivity. For example, if elastic cross sections could be measured down to $\approx 10^{-3}$ pb, then $\tilde{z}_{min}$ could be fully probed up to ≈ 0.17. Now, $\tilde{z}_{min}$ can be computed from Eq. (64) and be plotted against, say $m_{\tilde{e}_R}$, as shown in the bottom row of Fig. 28. For the example given above ($\tilde{z}_{min} \lesssim 0.17$) one could indirectly probe $\tilde{e}_R$ masses as high as ≈ 108 GeV. Note that a useful constraint on $m_{\tilde{e}_R}$ is possible because the correlation among the various sparticle masses in this model makes these scatter plots be rather well defined. This indirect experimental exploration still requires the identification of elastic supersymmetric events with $eX + \not{p}$ signature (in order to identify protons that contribute to the relevant z-distribution), but does not require a detailed reconstruction of each such event.

Figure 28: The most likely value of the relative proton energy loss in elastic processes (weighed by the various elastic cross sections) versus the total elastic cross section for selectron-neutralino production (top row) and $m_{\tilde{e}_R}$ (bottom row). The Leading Proton Spectrometer (LPS) will allow determination of $\tilde{z}_{min}$, and thus an indirect measurement of $m_{\tilde{e}_R}$.

11 Detailed calculations for Underground Labs and Underwater facilities

11.1 Gran Sasso and Super Kamiokande

The refinement on the calculation of the unification mass described in Sec. 6.2, to include two-loop effects and light supersymmetric thresholds, has a significant effect on the calculated value of the proton lifetime [43], since we take $M_H = 3M_U$. With the more accurate value of M_U we simply rescale our previously calculated τ_p values which satisfied $\tau_p^{(0)} > \tau_p^{exp}$, and find that $\tau_p^{(1)} = \tau_p^{(0)}[M_U^{(1)}/M_U^{(0)}]^2 > \tau_p^{exp}$ for only $\lesssim 25\%$ of the previously allowed points. The value of α_3 has a significant influence on the results since larger (smaller) values of α_3 increase (decrease) M_U, although the effect is more pronounced for low values of α_3. To quote the most conservative values of the observables, in what follows we take α_3 at its $+1\sigma$ value ($\alpha_3 = 0.126$). This choice of

Figure 29: The calculated values of the proton lifetime into $p \to \bar{\nu} K^+$ versus the lightest chargino (or second-to-lightest neutralino) mass for both signs of μ, using the more accurate value of the unification mass (which includes two-loop and low-energy supersymmetric threshold effects). Note that we have taken $\alpha_3 + 1\sigma$ in order to maximize τ_p. Note also that future proton decay experiments should be sensitive up to $\tau_p \approx 20 \times 10^{32}$ y.

α_3 also gives $\sin^2 \theta_w$ values consistent with the $\pm 1\sigma$ experimental range. Finally, in the search of the parameter space above, we considered only $\tan\beta = 2, 4, 6, 8, 10$ and found that $\tan\beta \lesssim 6$ was required. Our present analysis indicates that this upper bound is reduced down to $\tan\beta \lesssim 3.5$. Here we consider also $\tan\beta = 1.5, 1.75$ since low $\tan\beta$ maximizes $\tau_p \propto \sin^2 2\beta$. These add new allowed points (*i.e.*, $\tau_p^{(0)} > \tau_p^{exp}$) to our previous set, although most of them ($\gtrsim 75\%$) do not survive the stricter proton decay constraint ($\tau_p^{(1)} > \tau_p^{exp}$) imposed here.

In Fig. 29 we show the re-scaled values of τ_p versus the lightest chargino mass $m_{\chi_1^\pm}$. All points satisfy $\xi_0 \equiv m_0/m_{1/2} \gtrsim 6$ and $m_{\chi_1^\pm} \lesssim 150\,\mathrm{GeV}$, which are to be contrasted with $\xi_0 \gtrsim 3$ and $m_{\chi_1^\pm} \lesssim 225\,\mathrm{GeV}$ derived using the *weaker* proton decay constraint [42]. The upper bound on $m_{\chi_1^\pm}$ derives from its near proportionality to $m_{\tilde{g}}$, $m_{\chi_1^\pm} \approx 0.3 m_{\tilde{g}}$ [26, 42], and the result $m_{\tilde{g}} \lesssim 500\,\mathrm{GeV}$. The latter follows from the proton decay constraint $\xi_0 \gtrsim 6$ and the naturalness requirement $m_{\tilde{q}} \approx \sqrt{m_0^2 + 6 m_{1/2}^2} \approx \frac{1}{3} m_{\tilde{g}} \sqrt{6 + \xi_0^2} < 1\,\mathrm{TeV}$. With naturalness and H_3 mass assumptions, we then obtain [12]

$$\tau_p < 3.1\,(3.4) \times 10^{32}\,\mathrm{y} \quad \text{for} \quad \mu > 0\,(\mu < 0). \tag{65}$$

The $p \to \bar{\nu} K^+$ mode should then be readily observable at SuperKamiokande and Gran

[12]Note that in general, $\tau_p \propto M_{H_3}^2 [m_{\tilde{q}}^2 / m_{\chi_1^\pm}]^2 \propto M_{H_3}^2 [m_{\tilde{g}} (6 + \xi_0^2)]^2$ and thus τ_p can be made as large as desired by increasing sufficiently either the supersymmetric spectrum or M_H.

Sasso since these experiments should be sensitive up to $\tau_p \approx 2 \times 10^{33}$ y. Note that if M_H is relaxed up to its largest possible value consistent with low-energy physics, $M_H = 2.3 \times 10^{17}$ GeV [41], then in Eq. (65) $\tau_p \to \tau_p < 4.0\,(4.8) \times 10^{33}$ y, and only part of the parameter space of the model would be experimentally accessible. However, to make this choice of M_H consistent with high-energy physics (*i.e.*, $M_H < 2M_V$) one must have $M_V/M_\Sigma > 42$.

11.2 Neutrino Telescopes

The basic idea is that the neutralinos χ (lightest linear combination of the superpartners of the photon, Z-boson, and neutral Higgs bosons), which are weakly interacting massive particles (WIMPs), are assumed to make up the dark matter in the galactic halo —an important assumption which should not be overlooked— and can be gravitationally captured by the Sun or Earth [102, 103], after losing a substantial amount of energy through elastic collisions with nuclei. The neutralinos captured in the Sun or Earth cores annihilate into all possible ordinary particles, and the cascade decays of these particles as well as their interactions with the solar or terrestrial media produce high-energy neutrinos as one of several end-products. These neutrinos can then travel from the Sun or Earth cores to the vicinity of underground detectors, and interact with the rock underneath producing detectable upwardly-moving muons. Such detectors are rightfully called "neutrino telescopes", and the possibility of indirectly detecting various WIMP candidates has been considered in the past by several authors [104]. More recent analyses can be found in Refs. [105, 106, 107, 108, 109, 110, 111]. The calculation of the upwardly-moving muon fluxes induced by the neutrinos from the Sun and Earth in the still-allowed parameter space of the $SU(5)$ and $SU(5)\times U(1)$ supergravity models has been performed in Ref. [112]. The currently most stringent 90% C.L. experimental upper bounds, obtained at Kamiokande, for neutrinos from the Sun [113] and Earth [109] respectively, *i.e.*

$$\Gamma_{\rm Sun} < 6.6 \times 10^{-14}{\rm cm}^{-2}{\rm s}^{-1} = 2.08 \times 10^{-2}{\rm m}^{-2}{\rm yr}^{-1}, \tag{66}$$

$$\Gamma_{\rm Earth} < 4.0 \times 10^{-14}{\rm cm}^{-2}{\rm s}^{-1} = 1.26 \times 10^{-2}{\rm m}^{-2}{\rm yr}^{-1}. \tag{67}$$

Aiming at the next generation of underground experimental facilities, such as MACRO and other detectors at the Gran Sasso Laboratory [114], Super-Kamiokande [115], DUMAND, and AMANDA [116], where improvements in sensitivity by a factor of 2–100 are expected, we also delineate the region of the parameter space of these models that would become accessible with an improvement of experimental sensitivity by modest factors of two and twelve.

11.2.1 The capture rate

In order to calculate the expected rate of neutrino production due to neutralino annihilation, it is necessary to first evaluate the rates at which the neutralinos are captured in the Sun and Earth. Following the early work of Press and Spergel [102],

the capture of WIMPs by a massive body was studied extensively by Gould [103]. In our calculations, we make use of Gould's formula, and follow a procedure similar to that of Refs. [107, 108] in calculating the capture rate.

From Eq. (A10) of the second paper in Ref. [103], the capture rate of a neutralino of mass m_χ by the Sun or Earth can be written as

$$C = \left(\frac{2}{3\pi}\right)^{\frac{1}{2}} M_B \frac{\rho_\chi \bar{v}_\chi}{m_\chi} \sum_i \frac{f_i}{m_i} \sigma_i X_i, \tag{68}$$

where M_B is the mass of the Sun or Earth, ρ_χ and $\bar{v}_\chi$ are the local neutralino density and rms velocity in the halo respectively, σ_i is the elastic scattering cross-section of the neutralino with the nucleus of element i with mass m_i, f_i is the mass fraction of element i, and X_i is a kinematic factor which accounts for several important effects: (1) the motion of the Sun or Earth relative to Galactic center; (2) the suppression due to the mismatching of m_χ and m_i; (3) the loss of coherence in the interaction due to the finite size of the nucleus (see Ref. [103] for details).

In the summation in Eq. (68), we only include the ten most abundant elements for the Sun or Earth respectively, and use the mass fraction f_i of these elements as listed in Table A.1 of Ref. [107]. We choose $\bar{v}_\chi = 300\,\mathrm{km\,sec}^{-1}$, a value within the allowed range of the characteristic velocity of halo dark matter particles. To take into account the effect of the actual neutralino relic density, we follow the conservative approach of Ref. [107] for the local neutralino density ρ_χ: (a) $\rho_\chi = \rho_h = 0.3\,\mathrm{GeV/cm}^3$, if $\Omega_\chi h_0^2 > 0.05$; while (b) $\rho_\chi = (\Omega_\chi h_0^2/0.05)\rho_h$, if $\Omega_\chi h_0^2 \lesssim 0.05$. As for σ_i, the dominant contribution is the coherent interaction due to the exchange of two CP-even Higgs bosons h and H and squarks, and we use the expressions (A10) and (A11) of Ref. [108][13] to compute the spin-independent cross section for all the elements included. In addition, for capture by the Sun, we also evaluate the spin-dependent cross section due to both Z-boson exchange and squark exchange for the scattering from hydrogen according to Eq. (A5) (EMC model case) of Ref. [108]. It should be noted that in all these expressions the squarks were assumed to be degenerate. In the two supergravity models that we consider here this need not be the case, although for most of the parameter space this is a fairly good approximation. Hence, we simply use the average squark mass $m_{\tilde{q}}$ in this part of the calculation.

The kinematic factor X_i in Eq. (68), can be most accurately evaluated once the detailed knowledge of the mass density profile as well as the local escape velocity profile are specified for all the elements. In practice, this can be done by performing a numerical integration with the physical inputs provided by standard solar model or some sort of Earth model. Instead of performing such an involved calculation, we approximate the integral for each element by the value of the integrand obtained with the average effective gravitational "potential energy" ϕ_i times the integral volume. The values of ϕ_i are taken from Table A.1 of Ref. [107].

[13] We have corrected a sign error for the $H\chi\chi$ coupling in (A10) of Ref. [108].

11.2.2 The detection rate

We next describe the procedure employed by us to calculate the detection rate of upwardly-moving muons, resulting from the particle production and interaction subsequent to the capture and annihilation of neutralinos in the two supergravity models. The annihilation process normally reaches equilibrium with the capture process on a time scale much shorter than the age of the Sun or Earth. We assume this is the case, so that the neutralino annihilation rate equals half of the capture rate. The detection rate for neutrino-induced upwardly-moving muon events is then given by

$$\Gamma = \frac{C}{8\pi R^2}\sum_{i,F} D_i B_F \int \left(\frac{dN}{dE_\nu}\right)_{iF} E_\nu^2 dE_\nu. \tag{69}$$

In Eq. (69), D_i is a constant, R is the distance between the detector and the Sun or the center of the Earth, and $(dN/dE_\nu)_{iF}$ is the differential energy spectrum of neutrino type i as it emerges at the surface of the Sun or Earth due to the annihilation of neutralinos in the core of the Sun or Earth into final state F with a branching ratio B_F. It should be noted that in Eq. (69) that i is summed over muon neutrinos and anti-neutrinos, and that F is summed over final states that contribute to the high-energy neutrinos. The only relevant fermion pair final states are $\tau\bar{\tau}$, $c\bar{c}$, $b\bar{b}$, and (for the $SU(5)\times U(1)$ model) $t\bar{t}$ when $m_\chi > m_t$. The lighter fermions do not produce high-energy neutrinos since they are stopped by the solar or terrestrial media before they can decay [105].

The branching ratio B_F can be easily calculated as the relative magnitude of the thermal-averaged product of annihilation cross section into final state F (σ_F) with the Møller velocity v_M. Since the core temperatures of the Sun and Earth are very low compared with the neutralino mass ($T_{\rm Sun} \sim 1.34\times10^{-6}$ GeV; $T_{\rm Earth} \sim 4.31\times10^{-10}$ GeV), only the s-wave contributions are relevant, hence it is enough here to use the usual thermal average expansion up to zero-order of T/m_χ ($v_M \to 0$ limit), *i.e.*

$$B_F = \frac{\langle \sigma_F v_M\rangle}{\langle \sigma_{\rm tot} v_M\rangle} = \frac{a_F}{a_{\rm tot}}. \tag{70}$$

In Eq. (70), *all* kinematically allowed final states contribute to $a_{\rm tot}$. Besides all the fermion pair final states, we have also included boson pair final states WW, ZZ and hA in our calculation of B_F. Due to the parameter space constraints, these channels are not open for the $SU(5)$ model. But WW and ZZ channels are generally open in the $SU(5)\times U(1)$ model, and the hA channel also opens up for large values of $\tan\beta$ in the dilaton case. We should also remark that the annihilation channel into lightest CP-even higgs pair hh is always allowed kinematically in some portion of the parameter space for both supergravity models we consider, but since its s-wave contribution vanishes, we do not include it in the calculation of B_F. However, this channel is taken into account in the calculation of the neutralino relic density, which does affect the capture rate through the scaling of local density ρ_χ when $\Omega_\chi h_0^2 < 0.05$ (see Sec. 11.2.1). In addition, we have kept all the nonvanishing interference terms in the evaluation of $a_{\rm tot}$ and a_F.

The calculation of the neutrino differential energy spectrum is somewhat involved, since it requires a reasonably accurate tracking of the cascade of the particles which result from neutralino annihilation into each of the final state F. This involves the decay and hadronization of the various annihilation products and their interactions with the media of the Sun or the Earth's cores. In addition, at high energies, neutrinos interact with and are absorbed by solar matter, a fact that affects the spectrum. In Ref. [105], Ritz and Seckel rendered this calculation tractable by their adaptation of the Lund Monte Carlo for this purpose. Subsequently, analytic approximations to the Monte Carlo procedure outlined in their paper were refined and employed by Kamionkowski to calculate the neutrino energy spectra from neutralino annihilation for the MSSM in Ref. [108]. The procedure involved is described in detail in Ref. [112].

11.2.3 Results and Discussion

For each point in the parameter spaces of the supergravity models we consider, we have determined the relic abundance of neutralinos and then computed the capture rate in the Sun and Earth (as described in Sec. 11.2.1) and the resulting upwardly-moving muon detection rate (as described in Sec. 11.2.2). In Fig. 30, the predicted detection rates in the $SU(5)$ supergravity model are shown, based on the assumption that the mass of the triplet higgsino, which mediates dimension-five proton decay, obeys $M_{\tilde{H}_3} < 3M_U$. We have redone the calculation relaxing this assumption to $M_{\tilde{H}_3} < 10M_U$, in which case the results for the muon fluxes remain qualitatively the same, except that the parameter space is opened up somewhat. The dashed lines in Fig. 30 represent the current Kamiokande 90% C.L. upper limits Eqs. (66,67). Similarly, the predictions for $SU(5)\times U(1)$ supergravity are presented in Fig. 31 (32) for the no-scale (dilaton) scenario, again along with the Kamiokande upper limits (dashed lines). In Figs. 31,32, we have taken the representative value of $m_t =$ 150 GeV. Similar results are obtained for other values of m_t.

Several comments on these figures are in order. First, the kinematic enhancement of the capture rate by the Earth manifests itself in all figures as the big peaks near the Fe mass ($m_{\rm Fe} = 52.0$ GeV), as well as the smaller peaks around Si mass ($m_{\rm Si} = 26.2$ GeV). Second, there is a severe depletion of the rates near $m_\chi = \frac{1}{2}M_Z$ in Figs. 31,32, which is due to the decrease in the neutralino relic density. In the case of Earth capture, this effect is largely compensated by the enhancement near the Fe mass. As mentioned in Sec. 11.2.1, in our procedure, the relic density affects the local neutralino density ρ_χ only if $\Omega_\chi h_0^2 < 0.05$, while in the $SU(5)$ model this almost never happens, therefore, the effect of the Z-pole is not very evident in Fig. 30. Also, in Figs. 31,32, the various dotted curves correspond to different values of $\tan\beta$, starting from the bottom curve with $\tan\beta = 2$, and increasing in steps of two. These curves clearly show that the capture and detection rates increase with increasing $\tan\beta$, since the dominant piece of the coherent neutralino-nucleon scattering cross section via the exchange of the lightest Higgs boson h is proportional to $(1+\tan^2\beta)$. The capture rate decreases with increasing m_χ, since the scattering cross section falls off as m_h^{-4} and

Figure 30: The upwardly-moving muon flux in underground detectors originating from neutralino annihilation in the Sun and Earth, as a function of the neutralino mass in the $SU(5)$ supergravity model. The dashed lines represent the present Kamiokande 90% C.L. experimental upper limits.

m_h increases with increasing m_χ. It is expected that the detection rate in general also decreases for large value of m_χ, since it is proportional to the capture rate. However, the opening of new annihilation channels, such as the WW, ZZ and hA channels in $SU(5)\times U(1)$ supergravity, could have two compensating effects on the detection rate: (a) the presence of a new channel to produce high-energy neutrinos which leads to an enhancement of the detection rate; and (b) the decrease of the branching ratios for the fermion pair channels, which makes the neutrino yield from $\tau\bar{\tau}$, $c\bar{c}$ and $b\bar{b}$ smaller and hence reduces the detection rate. Therefore, these new annihilation channels could *either* increase *or* decrease the detection rate, depending which of these two effects wins over. We found that, for small values of $\tan\beta$ and $\mu < 0$, the WW channel can become dominant if open, basically because in this case the neutralino contains a rather large neutral wino component. This explains the distortion of the detection rate curves in the $\mu < 0$ half of Figs. 31 and 32. The effect of the ZZ channel turns out to be negligible in $SU(5)\times U(1)$ supergravity, since neutralinos with $m_\chi > M_Z$

Figure 31: The upwardly-moving muon flux in underground detectors originating from neutralino annihilation in the Sun and Earth, as a function of the neutralino mass in no-scale $SU(5)\times U(1)$ supergravity. The representative value of $m_t=150$ GeV has been used. The dashed lines represent the present Kamiokande 90% C.L. experimental upper limits.

Figure 32: Same as Fig. 31 but for dilaton $SU(5)\times U(1)$ supergravity.

have very small higgsino components. The same argument applies to the hA channel which sometimes opens up in the dilaton case for rather large values of $\tan\beta$.

In the dilaton case, for large values of $\tan\beta$ the CP-odd Higgs boson A can be rather light, and the presence of the A-pole when $m_\chi \sim \frac{1}{2}m_A$ makes the relic density very small. $\Omega_\chi h_0^2$ as a function of m_χ, is first lower than 0.05, it increases with m_χ, and eventually reaches values above 0.05, when neutralinos move away from the A-pole. Thus, the capture and detection rates also show this behavior, which can be seen as the few "anomalous" lines in Fig. 32. For lower values of $\tan\beta$, $\Omega_\chi h_0^2 < 0.05$, and there is no such effect. In the $SU(5)$ model, since the allowed points include different supersymmetry breaking scenarios and several values of m_t and $\tan\beta$, all these features are blurred. Nonetheless, in the same range of m_χ and for same values of $\tan\beta$ and m_t, we have found the results of these two models comparable, with the rates in the $SU(5)\times U(1)$ model slightly smaller due to the smaller relic density.

It is clear that at present the experimental constraints from the "neutrino telescopes" on the parameter space of the two supergravity models are quite weak. In fact, only the Kamiokande upper bound from the Earth can be used to exclude regions of the parameter space with $m_\chi \approx m_{\rm Fe}$ for both models, in particular for the $SU(5)\times U(1)$ model, due to the enhancement effect discussed above. However, it is our belief that the results presented in this paper will be quite useful in the future, when improved sensitivity in underground muon detection rates become available. An improvement in experimental sensitivity by a factor of two should be easily possible when MACRO [114] goes into operation, while a ten-fold improvement is envisaged when Super-Kamiokande [115] announces its results sometime by the end of the decade. More dramatic improvements in the sensitivity (by a factor of 20–100) may be expected from DUMAND and AMANDA [116], currently under construction. In addition, as recently argued [117], we think that perhaps the full parameter space of a large class of supergravity models, including the two specific ones considered here, may only be convincingly probed by a detector with an effective area of 1 km^2. It is interesting to note that, with a sensitivity improvement by a factor of 100, a large portion of the $\mu < 0$ half parameter space of the $SU(5)$ model can be probed. Unfortunately, the remaining portion, with fluxes below $\sim 10^{-4}$, can hardly be explored by underground experiments in the foreseeable future. For $SU(5)\times U(1)$ supergravity, in Figs. 33 and 34 we have plotted the allowed points in the $(m_{\chi_1^\pm}, \tan\beta)$ space. These points are those obtained originally in Refs. [68, 83], such that the neutrino telescopes constraint is also satisfied; no other constraints have been imposed. It can be seen here that the small voids of points for $m_{\chi_1^\pm} \approx 100$ GeV and a variety of values of $\tan\beta$ are excluded by the constraint from the "neutrino telescopes". In these figures we have marked by crosses the points in the $(m_{\chi_1^\pm}, \tan\beta)$ plane that MACRO should be able to probe (assuming an increase in the sensitivity by a factor of two) for the no-scale and dilaton scenarios respectively. As expected, the constraints from future "neutrino telescopes" will be strictest for large values of $\tan\beta$.

Figure 33: The allowed parameter space of no-scale $SU(5) \times U(1)$ supergravity (in the $(m_{\chi_1^\pm}, \tan\beta)$ plane) after the present "neutrino telescopes" (NT) constraint has been applied. Two values of m_t (130,150 GeV) have been chosen. The crosses denote those points which could be probed with an increase in sensitivity by a factor of two.

Figure 34: Same as Fig. 33 but for dilaton $SU(5) \times U(1)$ supergravity.

12 Detailed calculations for indirect experimental detection

12.1 Flavour Changing Neutral Current (FCNC) ($b \to s\gamma$)

There has recently been a renewed surge of interest on the flavor-changing-neutral-current (FCNC) $b \to s\gamma$ decay, prompted by the CLEO 95% CL allowed range [118]

$$\mathrm{BR}(b \to s\gamma) = (0.6 - 5.4) \times 10^{-4}. \tag{71}$$

Since the Standard Model (SM) prediction looms around $(2-5)\times 10^{-4}$ depending on the top-quark mass (m_t), a reappraisal of beyond the SM contributions has become topical [119, 120, 121, 122, 123]. We use the following expression for the branching ratio $b \to s\gamma$ [120]

$$\frac{\mathrm{BR}(b \to s\gamma)}{\mathrm{BR}(b \to ce\bar{\nu})} = \frac{6\alpha}{\pi} \frac{\left[\eta^{16/23}A_\gamma + \frac{8}{3}(\eta^{14/23} - \eta^{16/23})A_g + C\right]^2}{I(m_c/m_b)\left[1 - \frac{2}{3\pi}\alpha_s(m_b)f(m_c/m_b)\right]}, \tag{72}$$

where $\eta = \alpha_s(M_Z)/\alpha_s(m_b)$, I is the phase-space factor $I(x) = 1 - 8x^2 + 8x^6 - x^8 - 24x^4 \ln x$, and $f(m_c/m_b) = 2.41$ the QCD correction factor for the semileptonic decay. The A_γ, A_g are the coefficients of the effective $bs\gamma$ and bsg penguin operators evaluated at the scale M_Z. Their simplified expressions are given in the Appendix of Ref. [120], where the gluino and neutralino contributions have been justifiably neglected [124] and the squarks are considered degenerate in mass, except for the $\tilde{t}_{1,2}$ which are significantly split by m_t.

For the $SU(5)$ supergravity model we find [121]

$$2.3\,(2.6) \times 10^{-4} < \mathrm{BR}(b \to s\gamma)_{minimal} < 3.6\,(3.3) \times 10^{-4}, \tag{73}$$

for $\mu > 0\,(\mu < 0)$, which are all within the CLEO limits. One can show that $\mathrm{BR}(b \to s\gamma)$ would need to be measured with better than 20% accuracy to start disentangling the $SU(5)$ supergravity model from the SM. In Fig. 35 we present the analogous results for the $SU(5) \times U(1)$ supergravity cases: no-scale (top row) and strict no-scale (bottom row). The results are strikingly different than in the prior case. One observes that part of the parameter space is actually excluded by the new CLEO bound, for a range of sparticle masses. Perhaps the most surprising feature of the results is the strong suppression of $\mathrm{BR}(b \to s\gamma)$ which occurs for a good portion of the parameter space for $\mu > 0$. It has proven to be non-trivial to find a simple explanation for the observed cancellation. We discuss in Ref. [122] the implications of this indirect constraint on the prospects for direct experimental detection of these models.

Figure 35: The calculated values of BR($b \to s\gamma$) versus the chargino mass in no-scale (top row) and strict no-scale (bottom row, for the indicated values of m_t) $SU(5)\times U(1)$ supergravity. Note the fraction of parameter space excluded by the CLEO allowed range (in between the dashed lines).

12.2 Anomalous magnetic moment of the muon

The long standing experimental values of a_μ for each sign of the muon electric charge [125] can be averaged to yield [126]

$$a_\mu^{exp} = 1\ 165\ 923(8.5) \times 10^{-9}. \tag{74}$$

The uncertainty on the last digit is indicated in parenthesis. On the other hand, the various standard model contributions to a_μ have been estimated to be as follows [126]

$$\begin{aligned}
\text{QED}: &\quad 1\ 165\ 846\ 984(17)(28) \times 10^{-12} &(75)\\
\text{had.1}: &\quad 7\ 068(59)(164) \times 10^{-11} &(76)\\
\text{had.2}: &\quad -90(5) \times 10^{-11} &(77)\\
\text{had.3}: &\quad 49(5) \times 10^{-11} &(78)\\
\text{Total hadronic}: &\quad 7\ 027(175) \times 10^{-11} &(79)\\
\text{Electroweak}: &\quad 195(10) \times 10^{-11} &(80)
\end{aligned}$$

Here had.1 is the hadronic vacuum-polarization contribution to the second-order muon vertex, had.2 comes from higher-order hadronic terms, and had.3 from the hadronic light-by-light scattering. The total standard model prediction is then [126]

$$a_\mu^{SM} = 116\ 591\ 9.20(1.76) \times 10^{-9}. \tag{81}$$

Subtracting the experimental result gives [126]

$$a_\mu^{SM} - a_\mu^{exp} = -3.8(8.7) \times 10^{-9}, \tag{82}$$

which is perfectly consistent with zero. The uncertainty in the theoretical prediction is dominated by the uncertainty in the lowest order hadronic contribution (had.1), which ongoing experiments at Novosibirsk hope to reduce by a factor of two in the near future. This is an important preliminary step to testing the electroweak contribution, which is of the same order. The uncertainty in the experimental determination of a_μ is expected to be reduced significantly (down to 0.4×10^{-9}) by the new E821 Brookhaven experiment [127], which is scheduled to start taking data in late 1994. Any beyond-the-standard-model contribution to a_μ (with presumably negligible uncertainty) will simply be added to the central value in Eq. (82). Therefore, we can obtain an allowed interval for any supersymmetric contribution, such that $a_\mu^{susy} + a_\mu^{SM} - a_\mu^{exp}$ is consistent with zero at some given confidence level,

$$-4.9 \times 10^{-9} < a_\mu^{susy} < 12.5 \times 10^{-9}, \qquad \text{at } 1\sigma; \tag{83}$$

$$-10.5 \times 10^{-9} < a_\mu^{susy} < 18.1 \times 10^{-9}, \qquad \text{at } 90\%\text{CL}; \tag{84}$$

$$-13.2 \times 10^{-9} < a_\mu^{susy} < 20.8 \times 10^{-9}, \qquad \text{at } 95\%\text{CL}. \tag{85}$$

The supersymmetric contributions to a_μ have been computed to various degrees of completeness and in the context of several models, including the minimal supersymmetric standard model (MSSM) [128, 129, 130, 131, 132], an E_6 string-inspired model [133], and a non-minimal MSSM with an additional singlet [134, 135]. Because of the large number of parameters appearing in the typical formula for a_μ^{susy}, various contributions have often been neglected and numerical results are basically out of date. More importantly, a contribution which is roughly proportional to the ratio of Higgs vacuum expectation values ($\tan\beta$), even though known for a while [130, 131, 132, 135], has to date remained greatly unappreciated. This has been the case because in the past only small values of $\tan\beta$ were usually considered and the enhancement of a_μ^{susy}, which is the focus of this section, was not evident. In fact, such enhancement can easily make a_μ^{susy} run in conflict with the bounds given in Eq. (85), even after the LEP lower bounds on the sparticle masses are imposed. In Ref. [136] a reappraisal of this calculation has been done in the context of $SU(5)\times U(1)$ supergravity.

There are two sources of one-loop supersymmetric contributions to a_μ: (i) with neutralinos and smuons in the loop; and (ii) with charginos and sneutrinos in the loop. The general formula for the lowest order supersymmetric contribution to a_μ

has been given in Refs. [130, 131, 132, 135]. Here we use the expression in Ref. [135],

$$
\begin{aligned}
a_\mu^{susy} = -\frac{g_2^2}{8\pi^2}\Bigg\{ \sum_{\chi_i^0,\tilde{\mu}_j} \frac{m_\mu}{m_{\chi_i^0}} & \Big[(-1)^{j+1}\sin(2\theta) B_1(\eta_{ij}) \tan\theta_W N_{i1}[\tan\theta_W N_{i1} + N_{i2}] \\
& + \frac{m_\mu}{2M_W\cos\beta} B_1(\eta_{ij}) N_{i3} [3\tan\theta_W N_{i1} + N_{i2}] \\
& + \left(\frac{m_\mu}{m_{\chi_i^0}}\right)^2 A_1(\eta_{ij}) \Big\{ \tfrac{1}{4}[\tan\theta_W N_{i1} + N_{i2}]^2 + [\tan\theta_W N_{i1}]^2 \Big\} \Big] \\
- \sum_{\chi_j^\pm} \left[\frac{m_\mu m_{\chi_j^\pm}}{m_{\tilde{\nu}}^2} \frac{m_\mu}{\sqrt{2} M_W \cos\beta} B_2(\kappa_j) V_{j1} U_{j2} + \left(\frac{m_\mu}{m_{\tilde{\nu}}}\right)^2 \frac{A_1(\kappa_j)}{2} V_{j1}^2 \right] \Bigg\}. & \qquad (86)
\end{aligned}
$$

where N_{ij} are elements of the matrix which diagonalizes the neutralino mass matrix, and U_{ij}, V_{ij} are the corresponding ones for the chargino mass matrix, in the notation of Ref. [137]. Also,

$$
\eta_{ij} = \left[1 - \left(\frac{m_{\tilde{\mu}_j}}{m_{\chi_i^0}} \right)^2 \right]^{-1}, \qquad \kappa_j = \left[1 - \left(\frac{m_{\chi_j^\pm}}{m_{\tilde{\nu}}} \right)^2 \right]^{-1}, \qquad (87)
$$

and

$$
\begin{aligned}
B_1(x) &= x^2 - \tfrac{1}{2}x + x^2(x-1)\ln\left(\frac{x-1}{x}\right), & (88)\\
A_1(x) &= x^3 - \tfrac{1}{2}x^2 - \tfrac{1}{6}x + x^3(x-1)\ln\left(\frac{x-1}{x}\right), & (89)\\
B_2(x) &= -x^2 - \tfrac{1}{2}x - x^3\ln\left(\frac{x-1}{x}\right). & (90)
\end{aligned}
$$

As has been pointed out, the mixing angle of the smuon eigenstates is small (although it can be enhanced for large $\tan\beta$) and it makes the neutralino-smuon contribution suppressed. Moreover, the various neutralino-smuon contributions (the first three lines in Eq. (86)) tend to largely cancel among themselves [132].[14] This means that the chargino-sneutrino contributions (on the fourth line in Eq. (86)) will likely be the dominant ones. In fact, as we stress in this paper, the first chargino-sneutrino contribution (the "gauge-Yukawa" contribution) is enhanced relative to the second one (the "pure gauge" contribution) for large values of $\tan\beta$. This can be easily seen as follows.

Picturing the chargino-sneutrino one-loop diagram, with the photon being emitted off the chargino line, there are two ways in which the helicity of the muon can be flipped, as is necessary to obtain a non-vanishing a_μ:

[14]The original Fayet formula [128] is obtained from the third neutralino-smuon contribution in the limit of a massless photino and no smuon mixing.

(i) It can be flipped by an explicit muon mass insertion on one of the external muon lines, in which case the coupling at the vertices is between a left-handed muon, a sneutrino, and the wino component of the chargino and has magnitude g_2. It then follows that a_μ will be proportional to $g_2^2(m_\mu/\tilde{m})^2|V_{j1}|^2$, where $\tilde{m}$ is a supersymmetric mass in the loop and the V_{j1} factor picks out the wino component of the j-th chargino. This is the origin of the "pure gauge" contribution to a_μ^{susy}.

(ii) Another possibility is to use the muon Yukawa coupling on one of the vertices, which flips the helicity and couples to the Higgsino component of the chargino. One also introduces a chargino mass insertion to switch to the wino component and couple with strength g_2 at the other vertex. The contribution is now proportional to $g_2\lambda_\mu(m_\mu m_{\chi_j^\pm}/\tilde{m}^2)V_{j1}U_{j2}$, where U_{j2} picks out the Higgsino component of the j-th chargino. The muon Yukawa coupling is given by $\lambda_\mu = g_2 m_\mu/(\sqrt{2}M_W\cos\beta)$. This is the origin of the gauge-Yukawa contribution to a_μ^{susy}.

The ratio of the "pure gauge" to the "gauge-Yukawa" contributions is roughly then

$$g_2^2\,(m_\mu/\tilde{m})/(g_2\lambda_\mu) \sim g_2/\sqrt{1+\tan^2\beta}, \tag{91}$$

for $\tilde{m} \sim 100\,\mathrm{GeV}$. Thus, for small $\tan\beta$ both contributions are comparable, but for large $\tan\beta$ the "gauge-Yukawa" contribution is greatly enhanced.[15] This phenomenon was first noticed in Ref. [130]. It is interesting to note that an analogous $\tan\beta$ enhancement also occurs in the $b \to s\gamma$ amplitude [121], although its effect is somewhat obscured by possible strong cancellations against the QCD correction factor.

The results of the calculation in the no-scale and dilaton cases [136] are plotted in Fig. 36 respectively, against the gluino mass, for the indicated values of m_t. As anticipated, the values of $\tan\beta$ increase as the corresponding curves move away from the zero axis. Note that a_μ^{susy} drops off faster than naively expected (*i.e.*, $\propto 1/m_{\tilde{g}}$) since the U_{12} mixing element decreases as the limit of pure wino and Higgsino is approached for large $m_{\tilde{g}}$. Note also that a_μ^{susy} can have either sign, in fact, it has the same sign as the Higgs mixing parameter μ.[16] The incorrect perception that a_μ^{susy} is generally negative appears to be based on several model analyses where either μ was chosen to be negative or only some of the neutralino-smuon pieces were kept (which are mostly negative). Interestingly, the largest allowed values of $\tan\beta$ do not exceed the a_μ constraint since consistency of the models (*i.e.*, the radiative breaking constraint) requires larger gluino masses as $\tan\beta$ gets larger.

It is hard to tell what will happen when the E821 experiment reaches its designed accuracy limit. However, one point should be quite clear, the supersymmetric contributions to a_μ can be so much larger than the present hadronic uncertainty

[15] A similar enhancement in the second neutralino-smuon contribution is suppressed by small Higgsino admixtures (*i.e.*, $|N_{13}|, |N_{23}| \ll 1$).

[16] For comparison with earlier work, our sign convention for μ is opposite to that in Ref. [137].

Figure 36: The supersymmetric contribution to the muon anomalous magnetic moment in no-scale and dilaton $SU(5) \times U(1)$ supergravity, plotted against the gluino mass for the indicated values of m_t and $\tan\beta$ (which increase in steps of two). The dashed lines represent the 95%CL experimentally allowed range.

($\approx \pm 1.76 \times 10^{-9}$) that the latter is basically irrelevant for purposes of testing a large fraction of the allowed parameter space of the models. This is not true for the electroweak contribution and will also not hold for small values of $\tan\beta$. Should the actual measurement agree very well with the standard model contribution, then either $\tan\beta \sim 1$ or the sparticle spectrum would need to be in the TeV range. This situation is certainly a window of opportunity for sparticle detection before LEPII starts operating. Moreover, a significant portion of the explorable parameter space (those points with $m_{\chi_1^\pm} \gtrsim 100\,\mathrm{GeV}$ and equivalently $m_{\tilde{g}} \gtrsim 350\,\mathrm{GeV}$) is in fact beyond the reach of LEPII.

12.3 $\epsilon_{1,2,3,b}$

A complete study of one-loop electroweak radiative corrections in the supergravity models we consider here has been made in Refs. [138, 122, 139]. These calculations include some recently discovered important q^2-dependent effects, which occur when light charginos ($m_{\chi^\pm} \lesssim 60-70\,\mathrm{GeV}$) are present [140], and lead to strong correlations between the chargino and the top-quark mass. Specifically, one finds that at present the 90% CL upper limit on the top-quark mass is $m_t \lesssim 175\,\mathrm{GeV}$ in no-scale $SU(5) \times U(1)$ supergravity. These bounds can be strengthened for increasing chargino masses in the $50-100\,\mathrm{GeV}$ interval. For example, in$SU(5) \times U(1)$ supergravity, for $m_{\chi_1^\pm} \gtrsim 60\,(70)\,\mathrm{GeV}$, one finds $m_t \lesssim 165\,(160)\,\mathrm{GeV}$. As expected, the heavy sector of both models decouples quite rapidly with increasing sparticle masses, and at present, only ϵ_1 leads to constraints on the parameter spaces of these models. For future reference, it is important to note that global SM fits to all of the low-energy and electroweak data [56] constrain $m_t < 194, 178, 165\,\mathrm{GeV}$ for $m_{H_{SM}} = 1000, 250, 50\,\mathrm{GeV}$ at the 90% CL respectively.

One can show that an expansion of the vacuum polarization tensors to order q^2, results in three independent physical parameters. In the first scheme introduced to study these effects [141], namely the (S, T, U) scheme, a SM reference value for $m_t, m_{H_{SM}}$ is used, and the deviation from this reference is calculated and is considered to be "new" physics. This scheme is only valid to lowest order in q^2, and is therefore not applicable to a theory with new, light ($\sim M_Z$) particles. In the supergravity models we consider here, each point in parameter space is actually a distinct model, and a SM reference point is not meaningful. For these reasons, in Ref. [138] the scheme of Refs. [142, 140] was chosen, where the contributions are *absolute* and valid to higher order in q^2. This is the so-called $\epsilon_{1,2,3}$ scheme. More recently, a new observable (ϵ_b), which parametrizes the one-loop vertex corrections to the $Z \to b\bar{b}$ vertex, has been added to this set [143, 144]. Regardless of the scheme used, all of the global fits to the three physical parameters are *entirely consistent* with the SM at 90% CL.

With the assumption that the dominant "new" contributions arise from the process-independent (*i.e.*, "oblique") vacuum polarization amplitudes, one can combine several observables in suitable ways such that they are most sensitive to new

effects. It is important to note that not all observables can be included in the experimental fits which determine the $\epsilon_{1,2,3}$ paremeters, if only the oblique contributions are kept [138]. The most important non-oblique corrections are encoded in ϵ_b

It is well known in the MSSM that the largest contributions to ϵ_1 (*i.e.*, $\delta\rho$ if q^2-dependent effects are neglected) are expected to arise from the $\tilde{t}$-$\tilde{b}$ sector, and in the limiting case of a very light stop, the contribution is comparable to that of the t-b sector [145]. The remaining squark, slepton, chargino, neutralino, and Higgs sectors all typically contribute considerably less. For increasing sparticle masses, the heavy sector of the theory decouples, and only SM effects with a *light* Higgs survive. However, for very light chargino, a Z-wavefunction renormalization threshold effect can introduce a substantial q^2-dependence in the calculation, thus modifying significantly the standard $\delta\rho$ results. For completeness, in Ref. [138] the complete vacuum polarization contributions from the Higgs sector, the supersymmetric chargino-neutralino and sfermion sectors, and also the corresponding contributions in the SM were included.

In Fig. 37 we show the calculated values of ϵ_1 versus the lightest chargino mass ($m_{\chi_1^\pm}$) for the sampled points in the $SU(5)$ model and in no-scale $SU(5)\times U(1)$) supergravity. In the no-scale $SU(5)\times U(1)$ case three representative values of m_t were used, $m_t = 100, 130, 160\,\mathrm{GeV}$, whereas in the $SU(5)$ case several other values for m_t in the range $90\,\mathrm{GeV} \le m_t \le 160\,\mathrm{GeV}$ were sampled. In both models, but most clearly in the no-scale model one can see how quickly the sparticle spectrum decouples as $m_{\chi_1^\pm}$ increases, and the value of ϵ_1 asymptotes to the SM value appropriate to each value of m_t and for a *light* ($\sim 100\,\mathrm{GeV}$) Higgs mass. The threshold effect of $\chi_1^\pm$ is manifest as $m_{\chi_1^\pm} \to \frac{1}{2}M_Z$ and is especially visible for $\mu < 0$ in both models. The calculation of this effect is not expected to be very accurate as $m_{\chi_1^\pm} \to \frac{1}{2}M_Z$. However, according to Ref. [140], for $m_{\chi_1^\pm} > 50\,\mathrm{GeV}$, this estimate agrees to better than 10% with the result obtained in a more accurate way.

Recent values for $\epsilon_{1,2,3}$ obtained from a global fit to the LEP (*i.e.*, Γ_l, $A_{FB}^{l,b}$, A_{pol}^τ) and M_W/M_Z measurements are [146],

$$\epsilon_1 = (-0.9 \pm 3.7)10^{-3}, \quad \epsilon_2 = (9.9 \pm 8.0)10^{-3}, \quad \epsilon_3 = (-0.9 \pm 4.1)10^{-3}. \tag{92}$$

For ϵ_1 it is clear that virtually all the sampled points in the $SU(5)$ supergravity model are within the $\pm 1.64\sigma$ (90% CL) bounds (denoted by the two horizontal solid lines in the figures). Since several values for $90 \le m_t \le 160\,\mathrm{GeV}$ were sampled, the trends for fixed m_t are not very clear from the figure. Nonetheless, the points just outside the 1.64σ line correspond to $m_t = 160\,\mathrm{GeV}$, which are therefore excluded at the 90% CL. In the no-scale model, the upper bound on m_t depends sensitively on the chargino mass. For example, for $m_t = 160\,\mathrm{GeV}$, only light chargino masses would be acceptable at 90% CL. In fact, in Ref. [138] the region $130\,\mathrm{GeV} \le m_t \le 190\,\mathrm{GeV}$ was scanned in increments of $5\,\mathrm{GeV}$ and obtained the maximum values for $m_{\chi_1^\pm}$ allowed by the experimental value for ϵ_1 at 90% CL. There is a strong correlation between m_t and $m_{\chi_1^\pm}$: as m_t rises, the upper limit to $m_{\chi_1^\pm}$ falls, and vice versa. In particular, for $m_t \le 150\,\mathrm{GeV}$ all values of $m_{\chi_1^\pm}$ are allowed, while one could have m_t as large as $160\,(175)\,\mathrm{GeV}$ for $\mu > 0\,(\mu < 0)$ if the chargino mass were light enough.

Figure 37: The total contribution to ϵ_1 as a function of the lightest chargino mass $m_{\chi_1^\pm}$ for the $SU(5)$ model (upper row) and the no-scale $SU(5)\times U(1)$ model (bottom row). Points between the two horizontal solid lines are allowed at 90% CL. The three distinct curves (from lowest to highest) in the no-scale case correspond to $m_t = 100, 130, 160\,\mathrm{GeV}$.

The calculated values of ϵ_b are shown in Fig. 38 in conjuction with the values of ϵ_1, since these two observables are the only ones which constrain supersymmetric models at present. The 1-σ experimental ellipse is taken from Ref. [147]. Clearly smaller values of m_t fit the data better. The regions of the plot where the points accumulate correspond to the Standard Model limit, with a light Higgs boson.

13 The problem of mass and m_t

13.1 Generalities

The origin of mass is one of the most profound problems in Physics. Modern Field Theories try to answer this problem by invoking spontaneous broken gauge symmetry through the vacuum expectation value (vev) of an elementary (or composite) scalar field, the Higgs field. In general the masses of all particles (scalars, fermions, gauge

Figure 38: The correlated predictions for the ϵ_1 and ϵ_b parameters in units of 10^{-3} in the no-scale and dilaton $SU(5)\times U(1)$ supergravity scenarios. The ellipses represent the 1σ, 90%CL, and 95%CL contours obtained from all LEP data. The values of m_t are as indicated.

bosons) are proportional to this (or these) vev(s). The proportionality coefficients are: the quartic couplings (λ) for the scalars, the Yukawa couplings (y) for the fermions, and the gauge couplings (g) for the gauge bosons. Therefore we have (in principle)

$$\begin{aligned} m_s &= \lambda \langle \mathrm{vev} \rangle , & (93)\\ m_f &= y \langle \mathrm{vev} \rangle , & (94)\\ m_g &= g \langle \mathrm{vev} \rangle . & (95) \end{aligned}$$

The above general picture looks convincingly simple, but its implementation in realistic models is cumbersome. There are several reasons that prevent us from a complete and satisfactory solution, at present, of the mass problem. The quark and lepton mass spectrum, neglecting neutrinos, spans a range of at least five orders of magnitude $m_e = 0.5\,\mathrm{MeV}$ to $m_t \gtrsim 130\,\mathrm{GeV}$. If we take as "normal" the electroweak gauge boson masses, $\mathcal{O}(80-90)\,\mathrm{GeV}$, then a seemingly "heavy" top quark $\mathcal{O}(150\,\mathrm{GeV})$ looks perfectly normal, while all other quark and lepton masses look peculiarly small. Clearly, a natural theory cannot support fundamental Yukawa couplings extending over five orders of magnitude. The hope has always been that several of these Yukawa couplings are *naturally zero* at the classical level, and then quantum (radiative) corrections create a Yukawa coupling that reproduces reality. A modern version of this programm has arisen in string theory and will be discussed shortly. For the moment, let us emphasize that, despite the pessimism expressed above, certain features of the fermion mass spectrum has been already "explored" and have led to some spectacular predictions. Namely, in GUTs or SUSY-GUTs the difference between quark and lepton masses is attributed to the strong interactions (QCD) that make the quarks much heavier than the leptons (of the sa
me generation).

The successful prediction of the ratio $m_b/m_\tau \approx 2.9$ [49] has also led to the highly correlated prediction of $N_f = 3$, much before any primordial nucleosynthesis prediction. This was spectacularly confirmed at LEP $N_f = 2.980 \pm 0.027$ [1]. Another rather generic feature of supergravity GUTs, is their ability to trigger radiative spontaneous breaking of the EW-Symmetry [19, 20], thus explaining naturally why $m_W/m_{Pl} \approx 10^{-16}$. However, the triggering of this radiative breaking is only possible when the theory contains a Yukawa coupling of the order of "g", i.e. $y = \mathcal{O}(g)$, which is naturally identified with the top-quark Yukawa coupling. In other words, in supergravity GUTs it is not only that "heavy" t-quark is natural but it is needed, if we want to have a dynamical understanding of the EW-spontaneous breaking. Finally, string theory – more precisely its infrared limit – which encompasses naturally supergravity GUTs, is characterised by two features of relevance to us here (see *e.g.*, Refs. [32, 71]):

1. Most of the Yukawa couplings are naturally zero at the tree level and "pick up" non-zero values progressively at higher orders (through effective "non-renormalizable" terms), consistent with the problem of fermion masses observed in Nature.

2. Non-zero Yukawa couplings at tree level are automatically of $\mathcal{O}(g)$.

Once more a "heavy" top quark is a natural possibility and for the first time in string theory we may have a dynamical explanation of the origin of its large Yukawa coupling, *i.e.*, $\mathcal{O}(g)$.

13.2 A working ground to predict m_t

A "working ground" for further discussing and testing the above ideas is provided by $SU(5) \times U(1)$ supergravity, viewed as the infrared limit of string theory. The singularly unique properties of $SU(5) \times U(1)$ theory has been repeatedly emphasized above. Here we will concentrate on its features related to the specific purpose of predicting the best value for m_t. As a "descendant" of string theory, $SU(5) \times U(1)$ supergravity is characterized by two basic features:

a) A large top-Yukawa coupling: $\mathcal{O}(g)$.

b) The no-scale structure.

Notice that b) in conjunction with a), not only triggers EW-radiative breaking but, in principle, may also determine dynamically the magnitude of the SUSY breaking scale. As discussed above, the "allowable" "free parameters" are reduced to a "minimal" number, compared with any other model available at present. This unique feature of $SU(5) \times U(1)$ supergravity allows to put its "predictions" under severe experimental scrutiny. One of these "predictions" is the best value of m_t. This is obtained exploring the dependence on m_t of the "allowable" parameters space in $SU(5) \times U(1)$ supergravity, after all presently known constraints (theoretical, phenomenological, cosmological) have been duly taken into account, as described above. Interestingly enough, we find that the presently available constraints, once considered all together simultaneously (not one at a time) allow to put an upper bound on m_t [148]. This result is illustrated in Fig. 39. Note that for $m_t \gtrsim 190\,\text{GeV}$ no points in parameter space are allowed anyway because the top-quark Yukawa coupling would reach the Landau pole below the string scale [27].

This bottom-up approach indicates an m_t-range which is entirely consistent [149] with the top-bottom approach sketched above. This result is best appreciated in a plot of the top-quark Yukawa coupling at the string scale versus the top-quark mass, for various values of $\tan\beta$, as shown in Fig. 40. The dashed lines indicate typical string model-building predictions for the Yukawa coupling.

Figure 39: The number of allowed points in parameter space of no-scale and dilaton $SU(5) \times U(1)$ supergravity as a function of m_t when the basic theoretical and experimental LEP constraints have been imposed ("theory+LEP"), and when all known direct and indirect experimental constraints have been additionally imposed ("ALL").

14 Conclusions

We have reviewed the remarkable amount of work done during these last few years in order to disentangle one of the most fascinating problems we are confronted with: the existence of the Superworld. We know that new physics must exist beyond the Standard Model and the Superworld is the best candidate. On the other hand the hope to simplify the enormous variety of phenomena and have them describable in a theory with a very small number of parameters – possibly only one – can only be provided by String Theory. This is why we have chosen the two simplest possible supergravity models: $SU(5)$, a representative of field theory; and $SU(5) \times U(1)$, a representative of string theory. We have emphasized the relevance of performing detailed calculations in order to put the predictions under experimental tests with existing facilities. We have also pointed out that all experimental data have to be used simultaneously in order to constrain the parameter space of a theory. To appreciate the value of this procedure we show one example in Fig. 41 where all possible experimental constraints

Figure 40: The top-quark Yukawa coupling at the string scale in $SU(5)\times U(1)$ supergravity versus the top-quark mass for fixed values of $\tan\beta$ (larger values of $\tan\beta$ overlap with the $\tan\beta = 10$ curve). The dashed lines indicate typical string-like predictions for the Yukawa coupling.

are applied to the parameter space of $SU(5)\times U(1)$ supergravity in the $(m_{\chi_1^\pm}, \tan\beta)$ plane for $m_t = 150\,\mathrm{GeV}$ in both no-scale and dilaton scenarios.

The main properties of the two archetypical supergravity models discussed in this review are shown in Tables 6 and 7 for the $SU(5)$ supergravity model and $SU(5)\times U(1)$ supergravity, respectively.

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Figure 41: The parameter space of the no-scale and dilaton scenarios in $SU(5)\times U(1)$ supergravity in the $(m_{\chi_1^\pm}, \tan\beta)$ plane for $m_t = 150\,\mathrm{GeV}$. The periods indicate points that passed all constraints, the pluses fail the $B(b\to s\gamma)$ constraint, the crosses fail the $(g-2)_\mu$ constraint, and the diamonds fail the neutrino telescopes (NT) constraint. The dashed line indicates the direct reach of LEPII for chargino masses. Note that when various symbols overlap a more complex symbol is obtained.

Table 6: Major features of the $SU(5)$ supergravity model and its spectrum (All masses in GeV).

$\mathbf{SU(5)}$
• Not easily string-derivable, no known examples
• Symmetry breaking to Standard Model due to vevs of **24** and independent of supersymmetry breaking
• No simple doublet-triplet splitting mechanism
• Proton decay: $d = 5$ operators large, strong constraints needed
• Baryon asymmetry ?

Spectrum
• Parameters 5: $m_{1/2}, m_0, A, \tan\beta, m_t$
• Universal soft-supersymmetry breaking automatic
• $m_0/m_{1/2} > 3$, $\tan\beta \lesssim 3.5$
• Dark matter: $\Omega_\chi h_0^2 \gg 1$, 1/6 of points excluded
• $m_{\tilde{g}} < 400\,\mathrm{GeV}$, $m_{\tilde{q}} > m_{\tilde{l}} > 2m_{\tilde{g}} \gtrsim 500\,\mathrm{GeV}$
• $m_{\tilde{t}_1} > 45\,\mathrm{GeV}$
• $60\,\mathrm{GeV} < m_h < 100\,\mathrm{GeV}$
• $2m_{\chi^0_1} \approx m_{\chi^0_2} \approx m_{\chi^\pm_1} \approx 0.28 m_{\tilde{g}} \lesssim 100$
• $m_{\chi^0_3} \sim m_{\chi^0_4} \sim m_{\chi^\pm_2} \sim \lvert\mu\rvert$
• Chargino and Higgs easily accessible soon

Table 7: Major features of $SU(5)\times U(1)$ supergravity and a comparison of the two supersymmetry breaking scenaria considered. (All masses in GeV).

$\mathbf{SU(5)\times U(1)}$
• Easily string-derivable, several known examples
• Symmetry breaking to Standard Model due to vevs of $\mathbf{10},\overline{\mathbf{10}}$ and tied to onset of supersymmetry breaking
• Natural doublet-triplet splitting mechanism
• Proton decay: $d=5$ operators very small
• Baryon asymmetry through lepton number asymmetry (induced by the decay of heavy neutrinos) as processed by non-perturbative electroweak interactions

$\langle F_M\rangle_{m_0=0}$ (no-scale)	$\langle F_D\rangle$ (dilaton)
• Parameters 3: $m_{1/2},\tan\beta,m_t$	• Parameters 3: $m_{1/2},\tan\beta,m_t$
• Universal soft-supersymmetry breaking automatic	• Universal soft-supersymmetry breaking automatic
• $m_0=0$, $A=0$	• $m_0=\frac{1}{\sqrt{3}}m_{1/2}$, $A=-m_{1/2}$
• Dark matter: $\Omega_\chi h_0^2<0.25$	• Dark matter: $\Omega_\chi h_0^2<0.90$
• $m_{1/2}<475\,\mathrm{GeV}$, $\tan\beta<32$	• $m_{1/2}<465\,\mathrm{GeV}$, $\tan\beta<46$
• $m_{\tilde g}>245\,\mathrm{GeV}$, $m_{\tilde q}>240\,\mathrm{GeV}$	• $m_{\tilde g}>195\,\mathrm{GeV}$, $m_{\tilde q}>195\,\mathrm{GeV}$
• $m_{\tilde q}\approx 0.97 m_{\tilde g}$	• $m_{\tilde q}\approx 1.01 m_{\tilde g}$
• $m_{\tilde t_1}>155\,\mathrm{GeV}$	• $m_{\tilde t_1}>90\,\mathrm{GeV}$
• $m_{\tilde e_R}\approx 0.18 m_{\tilde g}$, $m_{\tilde e_L}\approx 0.30 m_{\tilde g}$ $m_{\tilde e_R}/m_{\tilde e_L}\approx 0.61$	• $m_{\tilde e_R}\approx 0.33 m_{\tilde g}$, $m_{\tilde e_L}\approx 0.41 m_{\tilde g}$ $m_{\tilde e_R}/m_{\tilde e_L}\approx 0.81$
• $60\,\mathrm{GeV}<m_h<125\,\mathrm{GeV}$	• $60\,\mathrm{GeV}<m_h<125\,\mathrm{GeV}$
• $2m_{\chi^0_1}\approx m_{\chi^0_2}\approx m_{\chi^\pm_1}\approx 0.28 m_{\tilde g}\lesssim 290$	• $2m_{\chi^0_1}\approx m_{\chi^0_2}\approx m_{\chi^\pm_1}\approx 0.28 m_{\tilde g}\lesssim 285$
• $m_{\chi^0_3}\sim m_{\chi^0_4}\sim m_{\chi^\pm_2}\sim\|\mu\|$	• $m_{\chi^0_3}\sim m_{\chi^0_4}\sim m_{\chi^\pm_2}\sim\|\mu\|$
• Spectrum easily accessible soon	• Spectrum accessible soon
• Strict no-scale: $B(M_U)=0$ $\tan\beta=\tan\beta(m_t,m_{\tilde g})$ $m_t\lesssim 135\,\mathrm{GeV}\Rightarrow\mu>0, m_h\lesssim 100\,\mathrm{GeV}$ $m_t\gtrsim 140\,\mathrm{GeV}\Rightarrow\mu<0, m_h\gtrsim 100\,\mathrm{GeV}$	• Special dilaton: $B(M_U)=2m_0$ $\tan\beta=\tan\beta(m_t,m_{\tilde g})$ $\tan\beta\approx 1.4-1.6$, $m_t<155\,\mathrm{GeV}$ $m_h\approx 61-91\,\mathrm{GeV}$

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